Term structures of discount rates: An international perspective

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Abstract

When a policy is evaluated, the rate at which future costs and benefits should be discounted depends upon their maturity and risk profile. When the shocks to the growth rate of consumption per capita are persistent, it is socially desirable to use a decreasing term structure of risk-free discount rates, and an increasing term structure of risk premia. We characterize these term structures when the representative agent has Epstein-Zin-Weil preferences and when log consumption follows an AR(1) process. We calibrate the model for 248 countries and economic zones of the World Bank database. We show that the efficient evaluation rules of long investment projects are very heterogeneous across countries. Using standard estimations of the preference parameters, the country-average 1-year and 20-year risk-free discount rates -1.42% and -3.27%. The 1-year and 20-year aggregate risk premia are respectively 4.21% and 7.12%. This study stresses both the necessity to use country-specific discount rates and the importance of estimating the risk profile of long-dated investment projects.

Keywords: Long-run risks, recursive preferences, long-termism, asset prices.

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1 Introduction

The choice of the discount rate is a key parameter to determine the priority list of investment projects or to evaluate a public policy for example, in particular when they have long-lasting benefits and costs. As explained by Baumol (1968), "*few topics in our discipline rival the social rate of discount as a subject exhibiting simultaneously a very considerable degree of knowledge and a very substantial level of ignorance.*" Five decades later, and after a passionate academic debate about the public discount rate (Groom and Hepburn (2016)) in relation to the shaping of climate policies, this topic remains non-consensual, as shown recently by Drupp et al. (2015). Policy evaluators, financial experts, and large investment institutions for example are still struggling with the concept and the way to implement efficient valuation procedures. The best illustration of this conflict is provided by the recent U.S. report (Interagency Working Group on Social Cost of Carbon (2015)), which left open the choice of the social discount rate, and in which the social cost of carbon has been estimated for three different discount rates: 2.5%, 3%, and 5%.

Public economists have used the Ramsey rule (Ramsey (1928)) to estimate the efficient risk-free discount rate. This rule is based on the idea that in a growing economy, investing for the future increases intertemporal/intergenerational inequalities. The discount rate net of the rate of pure preference for the present can then be interpreted as the minimum rate of return of a safe investment that compensate for this undesirable welfare impact of this investment. Under the Ramsey rule, this "wealth effect" is equal to the product of the aversion to intertemporal inequality of consumption and the growth rate of consumption. When this growth rate is uncertain, this rule must be extended to include a precautionary term. Under prudence (Drèze and Modigliani (1972), Kimball (1990)), consumers are willing to save more when the future becomes more uncertain. At the aggregate level, more precautionary investments should be implemented by reducing the risk-free discount rate. If social preferences are represented by the Discounted Expected Utility (DEU) with Constant Relative Risk Aversion (CRRA) and with a geometric Brownian motion to represent the dynamics of aggregate consumption, the so-called "extended Ramsey rule" reduces the risk-free discount rate by a precautionary term that is equal to half of the product of the square of the index of relative risk aversion and the annualized variance of the growth rate. This rule is well-known in the Consumption-based CAPM (CCAPM) literature. In a frictionless economy, this risk-free discount rate is the equilibrium interest rate. This rate can be maturity-dependent if the annualized mean or variance of the growth rate is maturity-dependent. For example, if shocks to the growth rate of consumption is persistent, then the macroeconomic uncertainty is magnified at long horizons. This makes the term structure of risk-free discount rates decreasing (Gollier (2008)).

Under risk aversion, it is socially desirable to penalize projects that increase the macroeconomic risk. This is usually done by adding a project-specific risk premium to the risk-free discount rate. In the DEU-CRRA-Brownian framework, this risk premium is proportional to the CCAPM beta of the project, which is defined as the elasticity of the dividend of the project to changes in aggregate consumption. As is well-known, the aggregate risk premium, i.e., the risk premium associated to an asset with a unit CCAPM beta, is equal to the product of relative risk aversion by the annualized variance of the growth rate of consumption. In the Brownian case, the annualized variance has a flat term structure, so the aggregate risk premium is maturity-independent. But if the shocks to growth are persistent, the annualized variance and the aggregate risk premium has an increasing term structure.

The previous two paragraphes establish the classical theory of discounting, as summarized for example by Gollier (2012). In the Brownian case, the calibration of the risk-free discount rate and of the aggregate risk premium requires two beliefs parameters (expectation and variance of growth), and two social preferences parameters (rate of preference for the present and relative risk aversion). This calibration faces a difficult challenge in the form of two puzzles. These puzzles are partly due to the fact that in the DEU model, the aversion to intertemporal inequality and risk aversion are identical. Given the large trend of growth in the U.S. during the last century, the CCAPM predicts an interest rate that is much larger than the observed 1% real interest rate on markets (risk-free rate puzzle, Weil (1989)). The theory is compatible with this observation only if one uses an aversion to intertemporal inequality below 1. But given the low volatility of growth, the CCAPM also predicts an equity premium that is much smaller than the equity premium of $5-6\%$ observed on U.S. markets during the last century (equity premium puzzle, Mehra and Prescott (1985)).[1] The theory is compatible with the data under the DEU-CRRA-Brownian model only if one uses an index of risk aversion much larger than unity. The implication of these puzzles for the cost-benefit analysis of investment projects is that this benchmark model tends to generate risk-free discount rates that are too large and risk premia which are too small. If one would apply these recommendations to public investments, the public sector would crowd out the private sector in the financing of the riskiest projects.

One possible road to solve these puzzles is to disentangle the aversion to intertemporal inequality from relative risk aversion, as proposed by Selden (1978)) and Epstein and Zin (1989) for example. EZW preferences allow for using a small aversion to intertemporal inequality to explain interest rates, and a large risk aversion to explain the equity premium puzzle. However, Weil (1989) showed that this so-called Epstein-Zin-Weil (EZW) model cannot solve the equity premium puzzle, because it requires an unrealistically large degree of risk aversion.[2] A more recent literature initiated by Bansal and Yaron (2004) has demonstrated that the two asset pricing puzzles can be solved by combining EZW preferences, a preference for an early resolution of uncertainty, and low-frequency fluctuations in consumption growth and its volatility. In this "long-run risks" literature, the representative agent is averse not only to the risk on consumption, but also to the risk on wealth. With low-frequency fluctuations, a lower growth rate next year does reduce consumption next year, but it also affects our beliefs about future growth. This reduces wealth, which is the market value of the flow of future consumption. This magnifies the short-term risk and has therefore the power to solve the two puzzles for the United States.

[1]The diversified portfolio of U.S. equity has a CCAPM beta around 3 (Bansal and Yaron (2004)).

[2]The EZW model is also referred to as the recursive utility model. Under these preferences, the aggregate risk premium continues to be equal to the product of the index of risk aversion by the variance of the growth rate, in the Brownian case. Because of the low variance of consumption growth observed in the U.S. data, a relative risk aversion around 40 continues to be necessary to explain the observed equity premium.

Most of models of asset pricing are calibrated on U.S. data. Capital and risks are not efficiently allocated in the world, so that the law of one price does not apply. This means in particular that risk-free discount rates and risk premia should be country-specific. In this paper, we attempt to adapt the ideas of the long-run risks literature to 248 countries and economic areas by using a database of the World Bank (WB) on regional GDP/cap over the period 1961-2015. As in Bansal and Yaron (2004), Bansal et al. (2012), Bansal et al. (2016), Beeler and Campbell (2012) and many others, we assume that the country-specific representative agent has EZW preferences. We also assume that the country-specific growth process is Auto-Regressive of order 1 (AR(1)).[3] The persistence coefficient of this AR(1) process is positive for 85% of the countries and economic zones contained in the WB database. This persistence implies that future consumption and wealth are positively correlated, as in the long-run risks literature.

We calibrate this model for each of the 248 economies under scrutiny. For each of them, we estimate the 3 parameters of the AR(1) process. We calibrate the preference parameters of the model by using the values that are standardly used in the long-run risks literature, with a rate of pure preference for the present of 1% per year, a relative aversion to intertemporal inequality of 2/3 (implying an elasticity of intertemporal substitution of 3/2), and a relative aversion to risk of 10. We provide an analytical solution to the term structures of interest rates and risk premia that is used to determine the pricing rules in each country. This yields two important results. First, the pricing rules that are efficient in the United Stated and for the members of the European Union are far to be representative of what should be done in most other countries. Because of either a smaller trend of growth, a larger volatility or a stronger persistence of shocks, many developing countries should use negative risk-free discount rates. They should also use much larger risk premia than in the Western world. Second, because of the persistence of the shocks, the term structures of risk-free discount rates and aggregate risk premia should be respectively decreasing and increasing. This reinforces the message that it is particularly crucial to measure the CCAPM beta of long-dated investment projects in order to determine their social value.

The paper is organized as follows. In Section 2, we solve analytically the asset pricing model with EZW preferences with an AR(1) growth process. In Section 3, we calibrate this model and we discuss the main policy recommendations that can be extracted from this exercise. Some concluding remarks are provided in Section 4.

2 The model

We consider a Lucas' fruit tree economy in which the consumption of fruits is exogenous. Consumption takes place in discrete time $t = 0, 1, ...$, with consumption per capita at date t being denoted c_t. We assume that the representative agent of the economy under scrutiny

[3] This is a simplification with respect to the the long-run risks literature in which it is the trend of growth which follows an AR(1), as in Bansal and Yaron (2004). Our specification implies much less long-term predictability compared to this literature. We do this because the estimation of the Bansal-Yaron model requires other financial data that are non-existent for many developing countries.

has recursive preferences. In the Epstein-Zin-Weil model, welfare V_t at date t is obtained by backward induction:

$$V_t^{1-\rho} = (1-\beta)c_t^{1-\rho} + \beta\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1-\rho}{1-\gamma}} \quad \text{if } \rho \neq 1 \tag{1}$$

$$\log V_t = (1-\beta)\log c_t + \beta \log\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \quad \text{if } \rho = 1. \tag{2}$$

where parameters γ and ρ are the indices of relative aversion to risk and to consumption fluctuations, respectively. Parameter $\beta = \exp(-\delta)$ is the discount factor, and δ is the rate of pure preference for the present. E_t is the expectation operator conditional to all information available at date t.

The current consumption c_0 of the representative agent is normalized to unity. Let $x_{t,t'} = \log(c_{t'}/c_t)$ denote the increase in log consumption between date t and date t'. It can also be interpreted as the continuously-compounded growth rate of consumption per capita between these two dates. The time unit is one year. We assume that $x_t = x_{t-1,t}$ follows an autoregressive process of order 1:

$$x_{t+1} = \mu + k(x_t - \mu) + \sigma\eta_{t+1}, \tag{3}$$

where μ is the unconditional expectation of the growth rate, σ^2 is the conditional variance of the growth rate, and $k \in \mathbb{R}$ is the coefficient of persistence of shocks to the growth rate, and η is standard Normal. Under this specification, log-consumption t periods ahead, defined as

$$x_{0,t} = \sum_{\tau=0}^{t-1} x_\tau = t\mu + (x_0 - \mu)k\frac{1-k^t}{1-k} + \sigma\sum_{\tau=0}^{t-1}\frac{1-k^{t-\tau}}{1-k}\eta_{\tau+1}, \tag{4}$$

is normally distributed with annualized mean and variance characterized by the following equations:

$$\frac{E_0[x_{0,t}]}{t} = \mu + (x_0 - \mu)k\frac{1-k^t}{t(1-k)} \tag{5}$$

$$\frac{Var_0[x_{0,t}]}{t} = \frac{\sigma^2}{(1-k)^2}\left(1 - 2\frac{k-k^{t+1}}{t(1-k)} + \frac{k^2-k^{2t+2}}{t(1-k^2)}\right). \tag{6}$$

When the current growth rate x_0 is equal to its unconditional mean μ, the expectations about future growth rate are neutral, in the sense that $E_0 x_{0,t}/t$ equals μ for all time horizons t. On the contrary, when the current growth rate x_0 is larger (smaller) than μ, expectations are diminishing (improving) in the sense that the annualized expected growth rate of consumption per capita is decreasing (increasing) in t.

The term structure of the annualized variance $Var[x_{0,t}]/t$ characterizes the accumulation of uncertainty at different maturities. In the benchmark case in which shocks to the growth rate are purely transitory, i.e., when k equals zero, this term structure is flat at $Var[x_{0,t}]/t = \sigma^2$. Suppose alternatively that there is some persistence to the shock to the growth rate of consumption, with $k \in [0,1[$. In that case, it is straightforward to verify that the annualized variance is increasing in the time horizon t. The positive serial correlation in annual growth rates magnifies the long-run uncertainty. Technically, $Var[x_{0,t}]/t$ goes from σ^2 to $\sigma^2/(1-k)^2$ for maturities go from one year to infinity.

We are interested in valuing at date 0 an asset or a policy that generates a single payoff c_t^ϕ at date t, for some $\phi \in \mathbb{R}$. Three specific risk profiles ϕ should be considered. Case $\phi = 0$ corresponds to a risk-free asset. Case $\phi = 1$ corresponds to a claim on aggregate consumption. Finally, Bansal and Yaron (2004) considered the case $\phi = 3$ as representative of a portfolio of diversified equity in the United States. As noticed by Campbell (1986), Abel (1999) and Martin (2013) for example, it is possible to interpret values $\phi > 1$ as a tractable way of modeling levered claims. If $P_0(\phi, t)$ denotes the equilibrium price of this asset at date 0, its annualized expected rate of return equals

$$R_{0,t}(\phi) = t^{-1} \log \left(\frac{E_0 c_t^\phi}{P_0(\phi, t)} \right). \tag{7}$$

The risk-free interest rate associated to maturity t is $r_{0,t}^f = R_{0,t}(0)$. The risk premium associated to risk profile ϕ and maturity t is defined as follows:

$$\pi_{0,t}(\phi) = R_{0,t}(\phi) - r_{0,t}^f. \tag{8}$$

It implies that the equilibrium price at date 0 of an asset delivering c_t^ϕ in t years is equal to its expected payoff $E_0 c_t^\phi$ discounted at its risk-adjusted discount rate $r_{0,t}^f + \pi_{0,t}(\phi)$. In this frictionless economy, the competitive equilibrium asset prices decentralizes an efficient allocation of capital. For example, the equilibrium interest rates provide the right signal to allocate risk-free capital. In other words, the equilibrium interest rates are the efficient discount rates to value risk-free investment projects and policies. A similar argument can be made for risk premia. This means that the term structures of interest rates and risk premia provides a full characterization of the way all investments projects, assets and public policies should be evaluated.

Except in the special cases of Discounted Expected Utility ($\rho = \gamma$, DEU) and of no-predictability ($k = 0$), determining the term structures of equilibrium interest rates and risk premia remains a complex matter. The derivation of the efficient discount rates is relegated to Appendix A, in which the following proposition is demonstrated.

Proposition 1. *Consider an economy in which the representative agent has recursive preferences (1)-(2) and in which log consumption follows the autoregressive process (3). In this economy, the equilibrium interest rates and risk premia can be approximated as follows:*

$$r_{0,t}^f = \delta + \rho \frac{E_0[x_{0,t}]}{t} - \frac{\rho^2}{2} \frac{Var_0[x_{0,t}]}{t} - \frac{(1-\rho)(\gamma-\rho)(1+b)^2}{2}\sigma^2 - \frac{\rho(\gamma-\rho)(1+b)}{1-k}\left(1 - \frac{k-k^{t+1}}{t(1-k)}\right)\sigma^2, \tag{9}$$

and $\pi_{0,t}(\phi) = \phi \pi_{0,t}(1)$ *with*

$$\pi_{0,t}(1) = \rho \frac{Var_0[x_{0,t}]}{t} + \frac{(\gamma-\rho)(1+b)}{1-k}\left(1 - \frac{k-k^{t+1}}{t(1-k)}\right)\sigma^2, \tag{10}$$

where b is the solution of the following equation:

$$b = (1+b)\beta k \exp\left((1-\rho)\left(\mu + \frac{1}{2}(1-\gamma)(1+b)\sigma^2\right)\right). \tag{11}$$

These approximations are exact when ρ equals 1 or γ.

We show in the appendix that when ρ equals 1, $\log(V_t/c_t)$ is linear in the state variable x_t, and $b = k\beta/(1-k\beta)$ is the multiplicative coefficient of this linear relationship. When ρ is not equal to 1, this linearity disappears, but the solution presented in Proposition 1 is based on the linear approximation of this relationship. Thus, coefficient b can be interpreted as the quasi elasticity of the intertemporal welfare per unit of current consumption (V_t/c_t) to a change in the current growth rate x_t. Notice that when $k \in]-1,1[$, b has the same sign as k.

Before discussing Proposition 1, we hereafter discuss two special cases: the DEU model $(\rho = \gamma)$, and the model without predictability $(k = 0)$.

2.1 Discounted Expected Utility

When the representative agent is an expected-utility maximizer with a CRRA utility function $v(c) = c^{1-\gamma}/(1-\gamma)$, the term structures of risk-free discount rates and risk premia are given by equations (9) and (10) with $\rho = \gamma$:

$$r^f_{0,t} = \delta + \gamma\frac{E_0[x_{0,t}]}{t} - \frac{\gamma^2}{2}\frac{Var_0[x_{0,t}]}{t} \tag{12}$$

$$\pi_{0,t}(\phi) = \phi\gamma\frac{Var_0[x_{0,t}]}{t}. \tag{13}$$

Interest rates have three terms in the right-hand side of equation (12): the rate of pure preference for the present, the wealth effect and the precautionary effect. In a growing economy, investing for the future has the undesirable effect to increase intertemporal inequalities. In a risk-free economy, the discount rate (net of δ) can be interpreted as the minimum internal rate of return of an investment project that compensates for this adverse effect. This wealth effect is proportional to the degree of aversion to intertemporal inequality $\rho = \gamma$ and to the degree of intertemporal inequality measured by $E_0[x_{0,t}]/t$. Limited to the two first terms in the right-hand side, equation (12) is called the "Ramsey rule" (Ramsey (1928)). Under uncertainty, the representative agent is willing to accumulate some precautionary savings. At equilibrium, this reduces interest rates, thereby inducing more "precautionary investments". The precautionary term is proportional to the annualized variance of log consumption. The purists would rather rewrite equation (12) as follows:

$$r^f_{0,t} = \delta + \gamma\frac{\log(E_0 c_t)}{t} - \frac{\gamma}{2}(1+\gamma)\frac{Var_0[x_{0,t}]}{t}. \tag{14}$$

The "pure" wealth term is the product of relative risk aversion and the growth rate of expected consumption. This leaves the "pure" precautionary term to be proportional to the product of relative risk aversion γ by the relative prudence $1+\gamma$.[4]

Risk premia are proportional to the risk profile ϕ of the investment. In fact, equation (13) corresponds to the classical Consumption-based CAPM risk premia where ϕ is the CCAPM beta of the project, and $\pi_{0,t}(1)$ are the systematic risk premia. They are proportional to risk aversion, and to the annualized variance of log consumption.

The term structures of interest rates and risk premia are easy to understand from this analysis. Suppose that the current growth rate x_0 equals its unconditional expectation μ, which implies that the expected annualized growth rate is μ for all maturities. Then, the term structures are univocally determined by the term structure of the annualized variance $Var_0[x_{0,t}]/t$. When shocks to growth exhibit some persistence $\rho \in]0,1[$, the annualized variance is increasing in t: Persistence magnifies the long-run risk of the economy. This implies a decreasing term structure of the risk-free discount rates. That tends to bias risk-free investments towards those which generate more distant benefits. On the other side, persistence also makes the term structure of systematic risk premia increasing. This is because investment projects whose risk profile duplicates the macroeconomic risk ($\phi = 1$) are made riskier by the persistence of macro shocks. Notice that the risk-adjusted discount rates are as follows:

$$r^f_{0,t} + \pi_{0,t}(\phi) = \delta + \gamma \frac{E_0[x_{0,t}]}{t} + \frac{\gamma}{2}(2\phi - \gamma)\frac{Var_0[x_{0,t}]}{t}. \tag{15}$$

The macroeconomic uncertainty reduces interest rates and it raises risk premia. The net effect on the risk-adjusted discount rate is positive if and only if the risk profile ϕ of the investment project is larger than half the degree of risk aversion γ. Under this condition, the term structure of risk-adjusted discount rates is increasing. When the current growth rate x_0 is larger than its historical mean μ, the term structure of interest rates is also affected by the diminishing expectations that the representative agent must have in this context, making longer interest rates smaller.

2.2 Geometric Brownian motion

The other limit case is when shocks to growth are purely transitory, so that k equals zero. This is a case in which future growth is unpredictable, and consumption per capita follows a geometric Brownian motion. In the absence of predictability, the intertemporal welfare V_t/c_t per unit of consumption must be independent of x_t. This implies that the linearization constant b must be equal to zero, as can be seen in equation (11). Moreover, $E_0[x_{0,t}]/t$ and $Var_0[x_{0,t}]/t$ simplify respectively to μ and σ^2 in that case. All in all, equations (9) and (10)

[4] Following Kimball (1990), relative prudence is defined as $-cv'''(c)/v''(c)$. Under CRRA, relative prudence is equal to relative risk aversion plus one. In the CRRA-Normal case examined here, the effect of risk on the interest rate is equivalent to a sure reduction of the growth rate of consumption by the relative precautionary premium, which is equal to the annualized variance multiplied by relative prudence. This explains equation (14).

can be rewritten in that case as follows:

$$r^f_{0,t} = \delta + \rho\left(\mu + \frac{\sigma^2}{2}\right) - \gamma(1+\rho)\frac{\sigma^2}{2} \tag{16}$$

$$\pi_{0,t}(\phi) = \phi\gamma\sigma^2. \tag{17}$$

In equation (16), the wealth effect is now proportional to ρ, which is the index of aversion to intertemporal inequality in the Epstein-Zin-Weil model. The precautionary term is proportional to the product of the index of risk aversion γ and the index of relative prudence $1+\rho$, in line with results obtained in a two-period model by Kimball and Weil (2009). Equation (17) is the classical CCAPM formula for risk premia. This means that this specification of the model cannot explain the equity premium puzzle. But by selecting a small ρ independently of risk aversion γ, this model can potentially solve the risk-free rate puzzle. Because there is no persistence, the term structures of interest rates and risk premia are flat.

2.3 Impact of persistence with recursive preferences

We first examine the impact of the persistence of shocks to growth on short-dated assets and investment projects. From equation (9), it is easy to verify that, conditional to x_0, the 1-year maturity interest rate in the general model equals

$$r^f_{0,1} = \delta + \rho\left(\mu + k(x_0-\mu) + \frac{\sigma^2}{2}\right) - \gamma(1+\rho)\frac{\sigma^2}{2} - (\gamma-\rho)b(2+(1-\rho)b)\frac{\sigma^2}{2}. \tag{18}$$

In expectation, x_0 equals μ, so that replacing x_0 by μ in the above equation generates the mean short-term interest rate in the economy. It is useful to compare this equation to equation (16), which prevails when $k=0$. The existence of persistence to shocks on growth does not affect the risk on log consumption $x_{0,1}$. However, because it affects the intertemporal welfare V_1 at date 1, it affects the pricing at date 0 of 1-year maturity assets in the recursive utility model. Its impact is measured by the last term in equation (18). The following proposition characterizes the impact of the persistence of shocks on the short interest rate. It is a direct consequence of the property that b is positive when the persistence coefficient k belongs to $]0,1[$.

Proposition 2. *Suppose that the persistence coefficient k is between 0 and 1, that ρ is smaller than 1, and that γ is larger than ρ. Then, the persistence of shocks to growth reduces the short interest rate $r^f_{0,1}$.*

Both assumptions $\rho < 1$ and $\gamma > \rho$ are classical in the long-run risks literature. In particular, condition $\gamma > \rho$ means that the representative agent has a Preference for an Early Resolution of Uncertainty (PERU). Under PERU and $\rho < 1$, the persistence of shocks reduces the short interest rate and can therefore help solving the risk-free rate puzzle. The uncertainty affecting the intertemporal welfare at date 1 plays the role of an additional source of risk which raises the willingness to accumulate precautionary saving at date 0, thereby reducing the equilibrium interest rate.

Turning to the short risk premium, equation (10) applied for $t = 1$ implies that

$$\pi_{0,1}(1) = \gamma\sigma^2 + (\gamma - \rho)b\sigma^2. \tag{19}$$

The first term in the right-hand side of this equality is the classical CCAPM systematic risk premium when shocks are purely transitory. This proves the following proposition.

Proposition 3. *Suppose that the persistence coefficient k is between 0 and 1, and that γ is larger than ρ. Then, the persistence of shocks to growth raises the short systematic risk premium $\pi_{0,1}(1)$.*

Although the persistence of shocks to growth does not affect the risk surrounding consumption at date 1, it affects the way one should penalize risky asset maturing at date 1. Under PERU, the persistence raises the systematic risk premium. This is because any asset that yields a payoff at date 1 that is positively correlated with the first period growth rate is also positively correlated to the first period intertemporal welfare, thereby increasing the global risk of the asset.

Remember that in the DEU framework examining in Section 2.1, the term structures of interest rate and risk premia re respectively decreasing and increasing. This is because the persistence of shocks magnifies long-run uncertainties. With recursive preferences, the timing of the resolution of the uncertainty may potentially modify this outcome. The term structures of $Er^f_{0,t}$ and $\pi_{0,t}(1)$ characterized by equations (9) and (10) are determined by the two functions $Var_0[x_{0,t}]/t$ and $1 - (k - k^{t+1})/(t(1-k))$. When k is between 0 and 1, these two functions are increasing in t. This proves the following proposition.

Proposition 4. *Suppose that the persistence coefficient k is between 0 and 1, and that γ is larger than ρ. Then, the term structures of the mean interest rates $Er^f_{0,t}$ and of the aggregate risk premia $\pi_{0,t}(1)$ are respectively decreasing and increasing.*

In other words, disentangling the aversions to risk and to intertemporal inequality cannot reverse the intuitive shapes of the term structures that prevail in the DEU framework. These theoretical predictions of this model are contradicted by asset prices observed on financial markets. In particular, a diversified portfolio of equities has a risk profile around $\phi = 3$ in developed countries, but recent findings document the fact that equity premia have a decreasing term structure (Binsbergen et al. (2012), Binsbergen and Koijen (2016), Belo et al. (2015), and Marfè (2016)).[5] A possible explanation is convincingly proposed by Marfè

[5] These findings are for maturities up to 10 years. For longer maturities, Giglio et al. (2015) and Giglio et al. (2016) provide evidence for real estate assets (leasehold contracts) with maturities measured in decades and centuries. Beeler and Campbell (2012) show evidence of mean-reversion rather than persistence in U.S. consumption growth in the period since 1930. Mean-reversion makes the aggregate risk in the longer run relatively smaller and can thus explain why interest rates and risk premia are respectively increasing and decreasing in maturity.

(2016). He argues that this comes from the fact that firms provide short-term insurance to their employees against the transitory fluctuations of their labor productivity, in line with the theory of implicit labor contract. This implies that the leverage of equity is larger in the short run than in the long run. The risk profile ϕ of equity has a decreasing term structure, thereby explaining the decreasing term structure of equity premia.

3 Calibration

We hereafter calibrate this model for 248 countries and economic zones identified in the World Bank national accounts data. We use the annual percentage growth rate of GDP per capita based on constant local currency. Aggregates are based on constant 2010 U.S. dollars. For each country or zone, we estimate the parameters (μ, σ, k) of the stochastic process (3). Most country data cover the period 1961-2015.

The preference parameters are based on the existing consensus of the long-run risks literature. The rate of pure preference for the present is assumed to be $\delta = 1\%$.[6] This means that one util delivered in 100 years is worth approximately one-third of a util today. We also assume that the relative risk aversion of the representative agent is 10. This means that the representative agent is willing to exchange an 50-50 chance of consuming forever 0.5 or forever 1.5 for a sure daily consumption of at least 0.54. In Table 1, we report the certainty equivalent consumption for other degrees of risk aversion. Although $\gamma = 10$ is compatible with observed asset prices as shown by Bansal and Yaron (2004), most economists consider that this degree of risk aversion remains unrealistically high.

Finally, we assume a relative aversion to intertemporal inequality of $\rho = 2/3$, which corresponds to an elasticity of intertemporal substitution of 1.5. This means that the representative agent is indifferent between consuming 0.5 in odd days and 1.5 in even days, or consuming 0.91 every day. In fact, Table 1 can alternatively interpreted as providing the constant consumption plan that is equivalent to the above-mentioned unequal consumption plan as a function of the degree of aversion to intertemporal inequality. Early studies by Hall (1985) and Hansen and Singleton (1983) suggested a value of ρ between 0 and 2, but this remains controversial. For example, in the debate among public and environmental economists that emerged after the publication of the Stern Review (Stern (2007)), all contributors to the debate used a degree of intertemporal inequality aversion between 1 and 4 to calibrate the Ramsey rule (see Gollier (2012)). However, the choice of a ρ smaller than unity is critically important in the long-run risks literature, in particular to generate a positive relationship between wealth and expected growth. Beeler and Campbell (2012) explains that the real interest rate is very volatile relative to predictable variation in consumption growth, thereby suggesting a ρ in the Ramsey rule that should be well above unity. Assuming a small ρ and a large γ implies a preference for an early resolution of uncertainty. Epstein et al. (2014) claim that the typical choice of the pair (γ, ρ) in the long-run risks literature implies an

[6] Bansal and Yaron (2004) assumed $\delta = 2.4\%$, whereas Bansal et al. (2016) assumed $\delta = 1.3\%$. Over the last century, many prominent economists have criticized the use of a positive rate of impatience when performing intergenerational welfare analyses. For a short discussion on this point, see for example Gollier (2012).

risk aversion	certainty equivalent
0.5	0.93
0.67	0.91
1	0.87
2	0.75
10	0.54
40	0.51

Table 1: Certainty equivalent consumption of a 50-50 chance of consuming either 0.5 forever, or 1.5 forever, as a function of relative risk aversion.

unrealistically large willingness to pay for an early resolution of uncertainty. In spite of all these critiques, the choice of $\gamma = 10$ and $\rho = 2/3$ remains a consensual calibration of the preferences with respect of risk and time. In this paper, we stick to this calibration.

In Appendix B, we provide the estimated value of parameters (μ, σ, k) for the 248 countries and economic zones contained in the data set of the World Bank. We also give information about the term structures of interest rates and risk premia for each of these economies. In Table 2, we selected a subset of these economies to illustrate some of the outcomes of this international analysis. In Appendix C, we draw the term structures of interest rates $Er^f_{0,t}$ and $\pi_{0,t}(1)$ for these 10 countries and economic zones. As a benchmark, let us first examine the case of the United States. The unconditional mean growth of GDP/cap has been $\mu = 2.08\%$ between 1961 and 2015, whereas the conditional volatility has been $\sigma = 1.89\%$. The persistence coefficient $k = 0.31$ is the smallest of this sample of 8 economies. Because of the low persistence of the shocks to growth, the term structure of interest rates are mostly flat, starting from $Er^f_{0,1} = 1.94\%$ for a one-year maturity, to $r^f_{0,\infty} = 1.79\%$. The 20-year interest rate is $Er^f_{0,20} = 1.80\%$. The term structure of aggregate risk premia is steeper, with $\pi_{0,1}(1) = 0.51\%$ and $\pi_{0,\infty}(1) = 0.74\%$. Suppose alternatively that one ignores the persistence of the shocks, so that $k = 0$ is assumed. Under this restriction of the model, one would estimate $\mu^* = 2.06\%$ and $\sigma^* = 2.02\%$. The term structures would be flat in that case, with an equilibrium risk-free rate $r^{f*} = 2.03\%$ and an equilibrium aggregate risk premium $\pi^*(1) = 0.41\%$. This is compatible with Propositions 2 and 3 which state that the persistence of shocks reduces the short interest rate and raises the short aggregate risk premium.[7]

Compared to the United States, France has a similar pair (μ, σ), but a much larger persistence coefficient $k = 0.57$. This generates steeper term structures, with a long interest rate going down to 1.43%, and a long aggregate risk premium going up to 1.20%. If we

[7]These propositions isolate the effect of $k > 0$, leaving σ unchanged. In this paragraph, we change k to zero, but σ is re-estimated under this restriction. This difference in the comparative static analysis is illustrated by China, for which the estimation of the volatility of growth is much larger when restricting the model to $k = 0$: $\sigma^* = 6.85\% > 4.37\% = \sigma$. This increased uncertainty makes the short interest rate $r^{f*} = 1.66\%$ smaller than $\bar{r}^f_{0,1} = 3.27\%$, and the aggregate risk premium $\pi^*(1) = 4.70\%$ larger than $\pi_{0,1}(1) = 2.99\%$ for this country.

Country	μ	σ	k	$\bar{r}^f_{0,1}$	$\bar{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
China	7.48	4.37	0.37	3.27	2.17	2.99	4.74	1.66	4.70
European Union	2.25	1.54	0.48	2.08	1.83	0.44	0.83	2.26	0.33
France	2.11	1.55	0.57	1.85	1.43	0.53	1.20	2.12	0.37
Latin America & Caribbean	1.73	2.10	0.40	1.49	1.20	0.72	1.19	1.73	0.54
Middle East & North Africa	1.76	3.20	0.46	0.43	-0.54	1.83	3.37	1.07	1.43
Nicaragua	0.47	5.49	0.36	-2.76	-4.32	4.53	7.00	-1.59	3.54
Sub-Saharan Africa	0.86	2.42	0.49	0.52	-0.11	1.09	2.08	0.89	0.79
United States	2.08	1.89	0.31	1.94	1.80	0.51	0.73	2.03	0.41
World	1.85	1.35	0.37	1.98	1.88	0.28	0.45	2.07	0.22
Zimbabwe	0.02	6.08	0.40	-4.40	-6.82	5.91	9.75	-2.69	4.49

Table 2: Summary statistics and term structures for a subset of 10 countries and economic zones. Parameters μ, σ and k are respectively the unconditional mean, the conditional volatility and the persistence coefficient of the annual growth rate of GDP/cap over the period 1961-2015. Variables $(\bar{r}^f_{0,1}, \bar{r}^f_{0,20}, \pi_{0,1}, \pi_{0,20})$ are the unconditional interest rate and risk premia associated to asset $\phi = 1$ for maturities $t = 1$ and $t = 20$. Finally, r^{f*} and π^* are respectively the interest rate and the risk premium for an asset $\phi = 1$ when the growth process is assumed to be i.i.d. ($k = 0$), and (μ, σ) are estimated accordingly. We assume that $\gamma = 10$, $\rho = 2/3$ and $\delta = 1\%$. All rates are in percent.

assume a risk profile of equity equaling $\phi = 3$ as in Bansal and Yaron (2004), this generates a long equity premium of 3.6%, in line with observed equity premia. China has a much larger unconditional mean growth rate ($\mu = 7.48\%$) and a much larger volatility ($\mu = 4.37\%$). The wealth effect dominates the precautionary effect, so that interest rates are larger at all maturities. The larger volatility unambiguously increases risk premia. At the opposite of the spectrum, the low historical trend of growth and large volatility in the Middle East and in Africa suggest using much lower interest rates, in particular for long maturities for which negative real risk-free discount rates should be recommended. On the contrary, as for China, large risk premia are socially desirable in these regions because of the intensity and persistence of the shocks to growth. Another extreme example is Zimbabwe, which had an almost non-existent growth over the last 5 decades ($\mu = 0.02\%$), and which at the same time faced high volatility ($\sigma = 6.08\%$) and relatively high persistence ($k = 0.4$). This yields very negative risk-free discount rates at all maturities ($\bar{r}^f_{0,1} = -4.40\%$ and $\bar{r}^f_{0,\infty} = -7.03\%$) and large aggregate risk premia ($\pi_{0,1} = 5.91\%$ and $\pi_{0,\infty} = 10.07\%$). It is vital for this country to invest in safe projects.

Notice that one should be cautious in using these price signals for economic zone that are not well integrated. The model is based on the assumption of a representative agent in each economy under scrutiny. In an economy with sizeable wealth inequalities, heterogenous preferences and idiosyncratic risks, such a representative agent exists only if credit and risk-sharing markets are complete and frictionless. This assumption is questionable even for an economically integrated country such as the United States. This assumption is certainly not

variable	mean	stdev	5%	50%	95%
x_0	1.30	3.55	-3.93	1.67	5.78
μ	2.20	1.75	-0.31	2.07	5.40
σ	4.31	3.47	1.54	3.47	8.83
k	0.30	0.27	-0.22	0.33	0.69
$\overline{r}^f_{0,1}$	-1.42	8.72	-8.26	0.71	2.81
$\overline{r}^f_{0,20}$	-3.27	13.70	-16.30	0.14	2.62
$\pi_{0,1}$	4.21	10.10	0.43	1.79	11.80
$\pi_{0,20}$	7.12	17.50	0.57	2.56	22.90

Table 3: Summary statistics for the 248 countries and economic zones of the World Bank database. All rates are in percent.

relevant for the European Union, as illustrated by the Greek crisis and the quasi-absence of European risk-sharing schemes. The problem is worse when contemplating other economic zones identified by the World Bank, such as Sub-Saharan Africa. Take the extreme example of "the World" which exhibits a very low volatility of growth ($\sigma = 1.35\%$), illustrating the fact that a large fraction of shocks to national economies are idiosyncratic and internationally diversifiable. This exercise shows that if these risks would be washed away through international risk-sharing, then investment projects should be evaluated with very low aggregate risk premia ($\pi_{0,1}(1) = 0.28\%$). But in reality, credit and risk-sharing markets are inefficient at the level of the world, and discount rates should be differentiated across countries to reflect differences in the prospects of growth and in macroeconomic uncertainties. The absence of the diversification of idiosyncratic country-specific risks implies a zero-mean background risk, which has the same impact as an increase in risk aversion (Gollier and Pratt (1996)).

In Table 3, we exhibit some summary statistics for the 248 economies of the database. As a complement, we draw in Figure 11 of Appendix C the histograms for the same macro-financial variables. The mean persistence is $k = 0.3$, with a 90% confidence interval of $[-0.22, 0.69]$. In fact, 85.4% of the countries and regions of the WB data base have a positive persistence coefficient. Short and long interest rates have a negatively skewed distribution. Although the median interest rates for 1-year and 20-year maturities are positive, their means are negative. In particular, the average short interest rate is -1.42%, and 5% of the countries of the database of a short interest rate below -8.26%. Term-specific aggregate risk premia are positively skewed, with realistic median values, but very large mean values. For example, the average 20-year aggregate risk premium equals 7.12%, and 5% of the countries contained in the WB database have a 20-year aggregate risk premium larger than 22.90%. In short, the "average country" of the world looks more like Nicaragua than like the United States in terms of efficient evaluation rules.

4 Concluding remarks

If a safe investment project is financed by diverting safe productive capital from other sectors of the economy, its implementation is socially desirable only if its internal rate of return is larger than the cost of safe capital in that economy. Because the interest rate measures this cost of capital, this is equivalent to requiring a positive value of future social net benefits discounted at the interest rate. Given the very low interest rates currently prevailing on financial markets, this simple arbitrage argument justifies using a very low discount rate, at least for maturities for which an interest rate can be observed on a liquid market. But in the Stern-Nordhaus controversy about the social cost of carbon, most experts used the Ramsey rule to estimate the social discount rate. The Ramsey rule identifies the social discount factor as the marginal rate of substitution between current and future consumption, using discounted expected utility as a representation of social preferences. To calibrate the Ramsey equation, Stern (2007) assumed an elasticity of marginal utility equaling 1 and an expected growth rate of consumption of 1.3%, yielding a social discount rate around 1.3%. This is not far from the market interest rates observed these days, and from the 2% median long-term (> 100 years) social discount rate recently recommended by 262 experts surveyed by Drupp et al. (2015). But Nordhaus (2008) and many others criticized this position because it imposes too much burden on current generations, in particular if this discount rate would be used for all projects. OMB (2003) and Nordhaus (2007) for example typically recommend a much larger discount rate, between 3% and 7%. This recommendation is based on the implicit assumption that investment projects have on average a CCAPM beta similar to a portfolio of diversified equity, so that the equity premium should be added to the risk-free discount rate to measure a risk-adjusted social discount rate. Over the last century, the equity premium has been between 2% and 6% in the Western world. But under expected utility, the relatively low volatility of growth in developed countries during that period implies a much smaller predicted equity premium, not larger 0.2% when using an elasticity of marginal utility of 1. This illustrates the equity premium puzzle.

This short description of the state of the art in discounting theory suggests that experts have a wide margin of interpretation about how to discount future costs and benefits when performing investment and public policy evaluations. This is a source of inefficiency. The absence of a settlement on this matter is partly due to the large discrepancy in the estimations of the elasticity of marginal utility depending upon its two different incarnations in the discounted expected utility model: The aversion to risk and the aversion to intertemporal inequality, the first being much larger than the second. Following recent developments in asset pricing theory, we disentangled these two preference parameters by using the Epstein-Zin-Weil recursive utility model to represent social preferences, and we took account of the persistence of shocks to growth to represent collective beliefs. We have shown that for majority of countries, these extensions of the classical welfare analysis tend to reduce the risk-free discount rate. The median risk-free discount rate is equal to 0.71% and 0.14%, for a maturity of respectively 1 year and 20 years. At the same time, it generates risk premia that are in line with observed asset prices. Indeed, the median aggregate risk premium is equal to 1.79% and 2.56%, for a maturity of respectively 1 year and 20 years.

Can we use this model to make recommendations about intergenerational discount rates, i.e., rates to be used when costs and benefits accrue to different generations? Following Harsanyi (1953), under the veil of ignorance, risk and time are perceived to be two equivalent concepts by the assembly of future generations. And under the independence axiom, their social welfare function should satisfy the additivity with respect to both risk and time. This justifies equalizing risk aversion and intertemporal inequality aversion in intergenerational redistribution contexts. We leave the problem of reconciling the intragenerational EZW preferences with intergenerational DEU preferences for future research.

References

ABEL, A. (1999): "Risk premia and term premia in general equilibrium," *Journal of Monetary Economics*, 43, 3–33.

BANSAL, R., D. KIKU, AND A. YARON (2012): "An Empirical Evaluation of the Long-Run Risks Model for Asset Prices," *Critical Finance Review*, 1, 183–221.

——— (2016): "Risks for the long run: Estimation with time aggregation," *Journal of Monetary Economics*, 82, 52–69.

BANSAL, R. AND A. YARON (2004): "Risks For the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481–1509.

BAUMOL, W. (1968): "On the social rate of discount," *American Economic Review*, 63, 788–802.

BEELER, J. AND J. CAMPBELL (2012): "The long-run risks model and aggregate asset prices: An empirical assessment," *Critical Finance Review*, 1, 141–182.

BELO, F., P. COLLIN-DUFRESNE, AND R. GOLDSTEIN (2015): "Endogenous dividend dynamics and the term structure of dividend strips," *Journal of Finance*, 70, 1115–1160.

BINSBERGEN, J. H. V., M. W. BRANDT, AND R. S. J. KOIJEN (2012): "On the timing of pricing of dividends," *American Economic Review*, 102, 1596–1618.

BINSBERGEN, J. H. V. AND R. S. J. KOIJEN (2016): "The Term Structure of Returns: Facts and Theory," *Journal of Financial Economics*, forthcoming.

CAMPBELL, J. (1986): "Bond and stock returns in a simple exchange model," *Quarterly Jounral of Economics*, 101, 785–804.

DRÈZE, J. AND F. MODIGLIANI (1972): "Consumption decisions under uncertainty," *Journal of Economic Theory*, 5, 308–335.

DRUPP, M., M. C. FREEMAN, B. GROOM, AND F. NESJE (2015): "Discounting disentangled: An expert survey on the determinants of the long-term social discount rate," Mimeo.

EPSTEIN, L., E. FARHI, AND T. STRZALECKI (2014): "How Much Would You Pay to Resolve Long-Run Risk?" *American Economic Review*, 104, 2680–2697.

EPSTEIN, L. AND S. ZIN (1989): "Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework," *Econometrica*, 57, 937–969.

GIGLIO, S., M. MAGGIORI, AND J. STROEBEL (2015): "Very long-run discount rates," *Quarterly Journal of Economics*, 130, 1–53.

GIGLIO, S., M. MAGGIORI, J. STROEBEL, AND A. WEBER (2016): "Climate change and long-run discount rates: Evidence from real estate," Harvard University.

Gollier, C. (2008): "Discounting with fat-tailed economic growth," *Journal of Risk and Uncertainty*, 37, 171–186.

——— (2012): *Pricing the Planet's Future: The Economics of Discounting in an Uncertain World*, Princeton University Press.

Gollier, C. and J. Pratt (1996): "Risk vulnerability and the tempering effect of background risk," *Econometrica*, 64, 1109–1124.

Groom, B. and C. Hepburn (2016): "Looking back at social discount rates: The influence of papers, presentations, political preconditions and personalities on policy," Mimeo.

Hall, R. E. (1985): "Real interest and consumption," NBER Working Paper 1694.

Hansen, L. and K. Singleton (1983): "Stochastic consumption, risk aversion, and the temporal behavior of asset returns," *Journal of Political Economy*, 91, 249–265.

Harsanyi, J. (1953): "Cardinal Utility in Welfare Economics and in the Theory of Risk-taking," *Journal of Political Economy*, 61, 434–435.

Interagency Working Group on Social Cost of Carbon, U. S. G. (2015): "Technical Support Document: Technical Update of the Social Cost of Carbon for Regulatory Impact Analysis Under Executive Order 12866," Tech. rep.

Kimball, M. (1990): "Precautionary savings in the small and in the large," *Econometrica*, 58, 53–73.

Kimball, M. and P. Weil (2009): "Precautionary saving and consumption smoothing across time and possibilities," *Journal of Money, Credit and Banking*, 41, 245–284.

Marfè, R. (2016): "Income insurance and the equilibrium terms-tructure of equity," *Journal of Finance*, forthcoming.

Martin, I. (2013): "Consumption-based asset pricing with higher cumulants," *Review of Economic Studies*, 80, 745–773.

Mehra, R. and E. C. Prescott (1985): "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145–161.

Nordhaus, W. D. (2007): "A review of the "Stern Review on the Economics of Climate Change"," *Journal of Economic Literature*, 686–702.

——— (2008): *A Question of Balance: Weighing the Options on Global Warming Policies*, Yale University Press.

OMB (2003): "Circular N. A-4 To the Heads of Executive Department Establishments, Subject: Regulatory Analysis," Tech. rep., Washington: Executive Office of the President.

Ramsey, F. (1928): "A mathematical theory of savings," *The Economic Journal*, 38, 543–559.

Selden, L. (1978): "A New Representation of Preferences over "Certain x Uncertain" Consumption Pairs: The Ordinal Certainty Equivalent," *Econometrica*, 46, 1045–1060.

Stern, N. (2007): *The Economics of Climate Change: The Stern Review,*, Cambridge University Press.

Weil, P. (1989): "The equity premium puzzle and the risk-free rate puzzle," *Journal of Monetary Economics*, 24, 401–421.

Appendix A: Proof of Proposition 1

Let us define $Z_\tau = V_\tau/c_\tau$ as the future expected utility per unit of current consumption. This variable plays a crucial role for the determination of the term structures. It is therefore important to characterize its main properties. Seen from date 0, Z_τ is uncertain for all $\tau \leq 1$. Because current growth rate x_τ can predict future growth, Z_τ is a function of x_τ. When $\rho = 1$, it happens that $z_\tau = \log(Z_\tau)$ is linear in this predictive variable. To see this, suppose that there exists a pair $(a, b) \in \mathbb{R}^2$ so that z_τ equals $a + bx_\tau$ for all τ, and let us verify that this guess solution solves backward-looking equation (2). We can rewrite this equation as follows:

$$\log Z_\tau = \beta \log \left(E_\tau \left(Z_{\tau+1} \frac{c_{\tau+1}}{c_\tau} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}. \tag{20}$$

Let us define operator χ in such a way that, for any random variable x,

$$\chi_\tau(x) = \log \left(E_\tau \exp(x) \right). \tag{21}$$

Then, we can rewrite equation (20) as

$$z_\tau = \frac{\beta}{1-\gamma} \chi_\tau \left((1-\gamma)(z_{\tau+1} + x_{\tau+1}) \right). \tag{22}$$

Using the AR(1) equation (3) and the linearity assumption for $z_{\tau+1}$, this is equivalent to

$$a + bx_\tau = \frac{\beta}{1-\gamma} \chi_\tau \left((1-\gamma)(a + (1+b)(\mu + k(x_\tau - \mu) + \sigma\eta_{\tau+1})) \right). \tag{23}$$

Because $\chi_\tau(q + x) = q + \chi_\tau(x)$ for all $q \in \mathbb{R}$, and because $\chi_\tau(q\eta_{\tau+1}) = 0.5q^2$, this dynamic equation has a solution with

$$b = \frac{k\beta}{1-k\beta} \tag{24}$$

and

$$a = \frac{(1+b)\beta}{1-\beta} \left((1-k)\mu + \frac{1}{2}(1-\gamma)(1+b)\sigma^2 \right). \tag{25}$$

When ρ is not equal to 1, there is no analytical solution for the relationship between z_τ and the state variable x_τ. However, as is standard in the literature on long-run risks, we will approximate this functional relationship by its linear approximation, using the linearization coefficient

$$b = \left. \frac{d\log(Z_\tau)}{dx_\tau} \right|_{x_\tau = \mu}. \tag{26}$$

Using equation (1), it can be shown that this is equivalent to condition (11). Notice that b is positive when k is in $[0, 1[$, which implies that the intertemporal welfare Z_τ is increasing in the current growth rate of consumption, as is intuitive when shocks are persistent.

Consider a marginal investment that costs p at date τ and that generates a payoff D at date $\tau + 1$. A simple marginalist argument implies that such an investment has no effect on intertemporal welfare V_τ at date τ if and only if

$$p = E_\tau \left[D \frac{S_{\tau+1}}{S_\tau} \right]. \tag{27}$$

where the one-period-ahead stochastic discount factor $S_{\tau+1}/S_\tau$ equals

$$\frac{S_{\tau+1}}{S_\tau} = \beta \left(\frac{c_{\tau+1}}{c_\tau} \right)^{-\gamma} Z_{\tau+1}^{\rho-\gamma} \left(E_\tau \left(\frac{c_{\tau+1}}{c_\tau} Z_{\tau+1} \right)^{1-\gamma} \right)^{\frac{\gamma-\rho}{1-\gamma}}. \tag{28}$$

Let us also define the log SDF $s_t = \log S_t$. This allows us to rewrite the pricing equations (28) as follows:

$$s_{\tau+1} - s_\tau = -\delta - \gamma x_{\tau+1} + (\rho - \gamma) z_{\tau+1} + \frac{\gamma - \rho}{1 - \gamma} \chi_\tau \left((1-\gamma)(x_{\tau+1} + z_{\tau+1}) \right). \tag{29}$$

Using the linearization of z_τ, equation (29) can be rewritten as

$$s_{\tau+1} - s_\tau = -\delta - \rho \left(\mu + k(x_\tau - \mu) \right) + \frac{1}{2} (\gamma - \rho)(1-\gamma)(1+b)^2 \sigma^2 + (\rho b - \gamma(1+b)) \sigma \eta_{\tau+1}. \tag{30}$$

This implies that

$$s_t - s_0 = -\delta t - \rho x_{0,t} + \frac{1}{2}(1-\gamma)(\gamma-\rho)(1+b)^2 \sigma^2 t + (\rho - \gamma)(1+b)\sigma \sum_{\tau=0}^{t-1} \eta_{\tau+1}. \tag{31}$$

Because preferences are recursive, one can use backward induction to determine equilibrium asset prices. Using backward induction, equation (27) implies that the price at date 0 of an asset that delivers a single payoff $D = c_t^\phi$ at date t must be equal to

$$P_0(\phi, t) = E_0 \left[c_t^\phi \frac{S_t}{S_0} \right]. \tag{32}$$

Let us define the annualized log return as $r_{0,t} = t^{-1} \log(c_t^\phi / P_0(\phi, t)$. The above pricing equation is then equivalent to:

$$\chi_0(Q) = 0, \tag{33}$$

with

$$Q = t r_{0,t} + s_t - s_0. \tag{34}$$

By definition of $r_{0,t}$, we have that

$$t r_{0,t} = t E_0 r_{0,t} + \phi(x_{0,t} - E_0 x_{0,t}) = t E_0 r_{0,t} + \phi \sigma \sum_{\tau=0}^{t-1} \frac{1 - k^{t-\tau}}{1-k} \eta_{\tau+1}. \tag{35}$$

This implies that

$$Q = t E_0 r_{0,t} - \delta t - \rho E_0 x_{0,t} + \frac{1}{2}(1-\gamma)(\gamma-\rho)(1+b)^2 \sigma^2 t + \sigma \sum_{\tau=0}^{t-1} \left((\phi - \rho) \frac{1-k^{t-\tau}}{1-k} (\rho - \gamma)(1+b) \right) \eta_{\tau+1}. \tag{36}$$

Because Q is Normal, the pricing equation (34) can be rewritten as follows:

$$E_0[Q] + \frac{1}{2}Var_0[Q] = 0, \tag{37}$$

with

$$E_0[Q] = tE_0 r_{0,t} - \delta t - \rho E_0 x_{0,t} + \frac{1}{2}(1-\gamma)(\gamma-\rho)(1+b)^2\sigma^2 t, \tag{38}$$

and

$$Var_0[Q] = \sigma^2 \sum_{\tau=0}^{t-1} \left((\phi-\rho)\frac{1-k^{t-\tau}}{1-k}(\rho-\gamma)(1+b) \right)^2. \tag{39}$$

These three equations imply that

$$tE_0 r_{0,t} = \delta t + \rho E_0 x_{0,t} - \frac{\sigma^2}{2}(1-\gamma)(\gamma-\rho)(1+b)^2 t - \frac{\sigma^2}{2}\sum_{\tau=0}^{t-1}\left((\phi-\rho)\frac{1-k^{t-\tau}}{1-k}(\rho-\gamma)(1+b) \right)^2. \tag{40}$$

Observe now that $tr_{0,t} = \log(c_t^{\phi}) - \log(P_0(\phi,t))$ is log-normally distributed, so that

$$tE_0 r_{0,t} = \log E_0[c_t^{\phi}] - \frac{1}{2}\phi^2 Var_0[x_{0,t}] - \log(P_0(\phi,t)) = tR_{0,t}(\phi) - \frac{1}{2}\phi^2 Var_0[x_{0,t}]. \tag{41}$$

This allows us to rewrite equation (40) as follows:

$$\begin{aligned} tR_{0,t}(\phi) = \delta t \quad &+ \quad \rho E_0 x_{0,t} - \frac{\sigma^2}{2}(1-\gamma)(\gamma-\rho)(1+b)^2 t + \frac{1}{2}\phi^2 Var_0[x_{0,t}] \\ &- \quad \frac{\sigma^2}{2}\sum_{\tau=0}^{t-1}\left((\phi-\rho)\frac{1-k^{t-\tau}}{1-k}(\rho-\gamma)(1+b) \right)^2. \end{aligned} \tag{42}$$

This is equivalent to

$$\begin{aligned} R_{0,t}(\phi) \quad &= \quad \delta + \rho\frac{E_0[x_{0,t}]}{t} - \left(\frac{\rho^2}{2} - \phi\rho\right)\frac{Var_0[x_{0,t}]}{t} \\ &- \quad (1-\rho)(\gamma-\rho)(1+b)^2\frac{\sigma^2}{2} - \frac{(\rho-\phi)(\gamma-\rho)(1+b)}{1-k}\left(1 - k\frac{1-k^t}{t(1-k)}\right)\sigma^2. \end{aligned} \tag{43}$$

Replacing ϕ by zero in the above equation yields the term structure of interest rates (9). Similarly, the term structure of risk premia $\pi_{0,t}(\phi)$ is obtained by subtracting $R_{0,t}(0)$ from $R_{0,t}(\phi)$. Using equation (43), we easily obtain that $\pi_{0,t}(\phi) = \phi\pi_{0,t}(1)$ where $\pi_{0,t}(1)$ is characterized by equation (10). ■

Appendix B: Detailed results for the 248 countries/regions

Summary statistics and term structures for the 248 countries and economic zones of the World Bank data base covering the period from 1961 to 2015. This table first documents the period used to estimate parameters μ, σ and k, which are respectively the unconditional mean, the conditional volatility and the persistence coefficient of the annual growth rate of GDP/cap. Variables $(\bar{r}^f_{0,1}, \bar{r}^f_{0,20}, \pi_{0,1}, \pi_{0,20})$ are the unconditional interest rate and risk premia associated to asset $\phi = 1$ for maturities $t = 1$ and $t = 20$. We also estimated the short interest rate $r^f_{0,1}$ conditional to the last available observation x_0 of the growth rate of GDP/cap. Finally, r^{f*} and π^* are respectively the interest rate and the risk premium for an asset $\phi = 1$ when the growth process is assumed to be i.i.d. $(k = 0)$, and (μ, σ) are estimated accordingly. We assume that $\gamma = 10$, $\rho = 2/3$ and $\delta = 1\%$. All rates are in percent.

Country	data	x_0	μ	σ	k	$\bar{r}^f_{0,1}$	$r^f_{0,1}$	$\bar{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Aruba	1995 – 2009	−5.72	−0.74	3.39	0.30	−0.90	−1.90	−1.34	1.60	2.29	−0.77	1.39
Andorra	1971 – 2013	4.43	0.02	2.64	0.62	−0.86	0.97	−2.55	1.73	4.39	0.06	1.14
Afghanistan	2003 – 2015	−1.29	4.26	5.84	−0.15	1.51	2.05	1.75	3.00	2.61	0.66	3.77
Angola	1986 – 2015	−0.27	1.74	6.80	0.57	−8.21	−8.98	−16.30	10.10	22.90	−3.81	7.11
Albania	1981 – 2015	2.72	2.64	7.25	0.42	−5.31	−5.29	−9.20	8.72	14.90	−2.70	6.58
Arab World	1976 – 2015	0.92	1.30	3.02	0.29	0.78	0.71	0.47	1.25	1.75	1.01	1.20
United Arab Emirates	1976 – 2015	2.38	−2.21	6.73	0.28	−5.77	−4.91	−7.24	6.10	8.44	−4.64	5.04
Argentina	1961 – 2014	−0.58	1.47	5.45	0.18	−1.04	−1.29	−1.55	3.59	4.41	−0.61	3.15
Armenia	1991 – 2015	2.61	4.47	9.60	0.50	−12.80	−13.40	−23.20	17.30	33.80	−8.07	14.00
Antigua and Barbuda	1978 – 2015	2.63	3.19	4.47	0.59	−1.70	−1.92	−5.64	4.61	10.80	0.52	3.13
Australia	1961 – 2015	0.89	1.95	1.68	0.16	2.02	1.91	1.98	0.33	0.40	2.03	0.30
Austria	1961 – 2015	0.05	2.35	1.83	0.31	2.15	1.67	2.01	0.48	0.69	2.27	0.39
Azerbaijan	1991 – 2015	−0.12	3.88	7.35	0.83	−34.80	−37.00	−97.10	25.80	123.00	−12.10	18.70
Burundi	1961 – 2015	−5.69	0.39	5.01	−0.03	−0.68	−0.57	−0.64	2.45	2.38	−1.43	3.01
Belgium	1961 – 2015	0.88	2.23	1.92	0.32	2.02	1.73	1.86	0.53	0.78	2.16	0.43
Benin	1961 – 2015	2.52	0.88	2.95	0.01	0.87	0.89	0.87	0.88	0.90	0.85	0.89
Burkina Faso	1961 – 2015	1.01	1.86	2.93	−0.17	1.67	1.77	1.74	0.74	0.63	1.50	0.90
Bangladesh	1961 – 2015	5.28	1.82	3.80	0.14	0.84	1.16	0.67	1.66	1.93	0.98	1.50
Bulgaria	1981 – 2015	3.63	2.53	3.81	0.54	−0.32	0.07	−2.43	3.00	6.32	0.96	2.12
Bahrain	1981 – 2015	1.78	0.51	3.69	0.26	−0.20	0.02	−0.59	1.79	2.40	−0.31	1.77
Bahamas, The	1961 – 2015	−0.07	0.92	5.89	0.36	−3.12	−3.36	−4.97	5.26	8.19	−1.76	4.10
Bosnia and Herzegovina	1995 – 2015	3.35	10.40	17.30	0.49	−43.40	−45.70	−75.20	53.80	104.00	−27.00	42.40
Belarus	1991 – 2015	−4.19	3.36	4.72	0.75	−7.10	−10.90	−20.10	8.06	28.30	−1.16	5.13
Belize	1961 – 2015	−0.23	2.70	3.49	0.47	0.68	−0.24	−0.52	2.22	4.12	1.47	1.59
Bermuda	1961 – 2013	−2.82	2.06	3.53	0.35	0.69	−0.46	0.05	1.88	2.89	1.17	1.45
Bolivia	1961 – 2015	2.38	1.17	3.19	0.36	0.40	0.69	−0.14	1.54	2.39	0.78	1.19
Brazil	1961 – 2015	−4.67	2.16	3.33	0.45	0.60	−1.45	−0.38	1.95	3.50	1.30	1.45
Barbados	1981 – 2015	0.69	0.73	2.56	0.41	0.50	0.49	0.05	1.07	1.79	0.73	0.84
Brunei Darussalam	1975 – 2015	−1.86	−0.60	5.78	0.15	−2.64	−2.76	−3.07	3.89	4.58	−2.39	3.53
Bhutan	1981 – 2015	1.95	5.40	4.40	−0.14	3.27	3.60	3.41	1.71	1.49	2.94	2.12

Country	data	x_0	μ	σ	k	$\bar{r}^f_{0,1}$	$r^f_{0,1}$	$\bar{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Botswana	1961 – 2015	−2.13	5.57	4.37	0.59	−0.05	−3.09	−3.96	4.51	10.70	2.24	2.95
Central African Republic	1961 – 2015	3.42	−1.22	6.22	−0.05	−2.74	−2.90	−2.63	3.70	3.51	−3.09	3.99
Canada	1961 – 2015	0.21	1.95	1.95	0.35	1.79	1.39	1.61	0.56	0.86	1.92	0.44
Central Europe	1991 – 2015	3.58	3.34	1.94	0.38	2.69	2.75	2.46	0.59	0.95	1.88	1.21
Switzerland	1981 – 2015	−0.29	0.95	1.60	0.31	1.31	1.05	1.20	0.37	0.53	1.39	0.29
Channel Islands	1999 – 2007	5.15	0.75	2.78	0.54	−0.09	1.49	−1.21	1.59	3.35	0.67	1.20
Chile	1961 – 2015	1.01	2.68	4.19	0.30	0.66	0.33	0.02	2.43	3.45	1.14	1.96
China	1961 – 2015	6.36	7.48	4.37	0.37	3.27	2.99	2.17	2.99	4.74	1.66	4.70
Cote d'Ivoire	1961 – 2015	5.84	0.37	4.76	0.29	−1.45	−0.41	−2.23	3.09	4.33	−0.83	2.57
Cameroon	1961 – 2015	3.64	0.93	5.17	0.30	−1.62	−1.09	−2.60	3.71	5.25	−0.90	2.99
Congo, Rep.	1961 – 2015	0.08	1.47	4.63	0.47	−1.76	−2.20	−3.92	3.91	7.31	−0.35	2.85
Colombia	1961 – 2015	2.15	2.25	1.95	0.29	2.04	2.02	1.90	0.53	0.74	2.15	0.42
Comoros	1981 – 2014	−0.36	−0.35	2.91	−0.26	0.25	0.25	0.34	0.69	0.54	0.01	0.94
Cabo Verde	1981 – 2015	1.22	4.92	4.09	0.40	1.77	0.79	0.65	2.72	4.50	2.60	2.06
Costa Rica	1961 – 2015	1.68	2.43	2.78	0.36	1.57	1.39	1.17	1.17	1.81	1.70	1.00
Caribbean small states	1967 – 2015	0.80	1.33	2.31	0.42	1.05	0.90	0.64	0.90	1.54	1.34	0.67
Cuba	1971 – 2013	2.51	2.46	5.17	0.51	−2.52	−2.51	−5.89	5.25	10.60	−0.44	3.77
Cyprus	1976 – 2015	2.91	2.87	2.92	0.56	1.02	1.04	−0.38	1.85	4.06	1.14	2.49
Czech Republic	1991 – 2015	3.94	2.26	2.46	0.32	1.74	2.10	1.49	0.87	1.27	0.87	1.53
Germany	1971 – 2015	1.15	1.87	1.97	0.10	1.90	1.85	1.87	0.43	0.48	1.93	0.40
Djibouti	1991 – 2014	4.59	0.43	2.12	0.73	−0.68	1.37	−3.07	1.56	5.29	−0.20	1.51
Dominica	1978 – 2015	2.28	2.74	4.99	−0.21	1.23	1.30	1.46	2.08	1.71	0.60	2.84
Denmark	1961 – 2015	0.60	1.82	2.22	0.21	1.69	1.52	1.59	0.62	0.78	1.80	0.55
Dominican Republic	1961 – 2015	5.71	3.28	4.80	0.02	1.30	1.33	1.27	2.35	2.40	1.01	2.49
Algeria	1961 – 2015	1.98	1.79	6.78	−0.17	−0.87	−0.89	−0.50	3.97	3.38	−2.51	5.39
East Asia & Pacific (ehi)	1961 – 2015	5.70	5.99	2.39	0.43	4.08	4.00	3.63	0.97	1.69	3.60	1.42
Early-demographic dividend	1961 – 2015	2.74	2.07	1.55	0.55	1.87	2.11	1.50	0.51	1.08	2.09	0.35
East Asia & Pacific	1961 – 2015	3.20	3.65	1.92	0.26	3.01	2.93	2.90	0.49	0.66	3.11	0.41
Europe & Central Asia (ehi)	1990 – 2015	−1.22	1.30	4.20	0.64	−3.12	−4.19	−7.81	4.55	11.90	−0.84	3.14
Europe & Central Asia	1971 – 2015	1.04	1.68	1.63	0.33	1.77	1.63	1.65	0.39	0.58	1.88	0.31

Country	data	x_0	μ	σ	k	$\bar{r}^f_{0,1}$	$r^f_{0,1}$	$\bar{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Ecuador	1961 – 2015	−1.21	1.68	2.76	0.33	1.13	0.49	0.78	1.11	1.66	1.35	0.89
Egypt, Arab Rep.	1966 – 2015	2.00	2.68	2.30	0.53	1.71	1.46	0.96	1.08	2.26	2.07	0.80
Euro area	1961 – 2015	1.41	2.29	1.64	0.55	1.94	1.62	1.51	0.58	1.25	2.21	0.42
Eritrea	1993 – 2011	6.41	0.91	6.09	0.39	−3.74	−2.31	−6.06	5.86	9.54	−2.61	5.61
Spain	1961 – 2015	3.35	2.51	1.82	0.69	1.49	1.87	0.20	1.01	3.02	2.05	0.88
Estonia	1996 – 2015	1.26	4.62	5.53	0.40	−0.49	−1.39	−2.56	4.97	8.24	0.94	3.86
Ethiopia	1982 – 2015	6.93	2.61	6.28	0.32	−2.30	−1.36	−4.00	5.69	8.39	−1.18	4.60
European Union	1961 – 2015	1.68	2.25	1.54	0.48	2.08	1.90	1.83	0.44	0.83	2.26	0.33
Fragile and conflict	1989 – 2015	−0.37	1.28	6.44	−0.31	−0.59	−0.25	−0.11	3.24	2.46	−2.46	4.96
Finland	1961 – 2015	0.17	2.38	2.77	0.44	1.34	0.69	0.69	1.33	2.36	1.80	1.01
Fiji	1961 – 2015	2.76	1.63	4.40	−0.12	0.73	0.64	0.84	1.74	1.56	0.40	2.00
France	1961 – 2015	0.68	2.11	1.55	0.57	1.85	1.31	1.43	0.53	1.20	2.12	0.37
Micronesia, Fed. Sts.	1987 – 2014	−3.71	0.67	2.98	0.00	0.74	0.74	0.74	0.89	0.89	0.63	0.93
Gabon	1961 – 2015	1.59	1.86	8.83	0.32	−7.60	−7.66	−10.90	11.10	16.40	−5.23	9.14
United Kingdom	1961 – 2015	1.50	2.01	1.92	0.37	1.83	1.71	1.63	0.57	0.89	1.98	0.43
Georgia	1966 – 2015	4.12	2.25	7.11	0.75	−19.30	−18.40	−47.50	17.40	61.50	−7.19	11.70
Ghana	1961 – 2015	1.52	0.98	3.96	0.39	−0.62	−0.48	−1.62	2.49	4.07	0.07	1.89
Guinea	1987 – 2015	−2.54	0.14	1.59	0.39	0.73	0.04	0.57	0.40	0.65	0.85	0.30
Gambia, The	1967 – 2014	−2.33	0.58	3.39	−0.09	0.55	0.73	0.61	1.06	0.97	0.35	1.20
Guinea-Bissau	1971 – 2015	2.31	0.60	6.60	−0.20	−1.42	−1.65	−1.03	3.67	3.04	−2.64	4.73
Equatorial Guinea	1981 – 2015	−14.70	11.80	25.30	0.42	−81.60	−89.10	−127.00	99.70	172.00	−58.20	80.30
Greece	1961 – 2015	0.40	2.16	3.47	0.55	−0.18	−0.84	−2.12	2.59	5.64	0.92	1.94
Grenada	1978 – 2015	2.95	2.84	4.32	0.15	1.09	1.10	0.86	2.17	2.54	1.29	2.00
Greenland	1971 – 2009	−5.40	1.97	4.41	0.18	0.33	−0.57	−0.00	2.36	2.89	0.56	2.29
Guatemala	1961 – 2015	2.06	1.33	1.89	0.56	1.10	1.37	0.51	0.78	1.71	1.45	0.53
Guyana	1961 – 2015	2.58	1.53	4.53	0.34	−0.65	−0.41	−1.59	3.00	4.50	0.06	2.35
High income	1961 – 2015	1.35	2.30	1.54	0.47	2.12	1.82	1.89	0.43	0.80	2.29	0.31
Hong Kong SAR, China	1966 – 2015	1.46	4.22	3.95	0.21	2.16	1.77	1.83	1.95	2.47	2.36	1.69
Honduras	1961 – 2015	2.18	1.44	2.74	0.23	1.14	1.25	0.95	0.96	1.26	1.24	0.82
Heavily indebted poor	1961 – 2015	2.00	0.64	1.60	0.57	0.85	1.36	0.40	0.56	1.26	1.06	0.40

Country	data	x_0	μ	σ	k	$\overline{r}^f_{0,1}$	$r^f_{0,1}$	$\overline{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Croatia	1996 – 2015	1.98	2.15	3.16	0.31	1.19	1.16	0.80	1.41	2.03	1.44	1.50
Haiti	1999 – 2015	0.33	−0.31	2.63	−0.10	0.30	0.25	0.34	0.63	0.57	0.21	0.75
Hungary	1992 – 2015	3.17	2.33	2.27	0.40	1.78	2.01	1.43	0.84	1.39	1.75	0.78
IBRD only	1961 – 2015	2.47	3.02	1.49	0.61	2.43	2.21	1.92	0.55	1.34	2.68	0.37
IDA & IBRD total	1961 – 2015	2.22	2.75	1.48	0.61	2.25	2.04	1.75	0.54	1.33	2.51	0.37
IDA total	1961 – 2015	2.33	1.43	2.46	0.42	1.02	1.27	0.57	1.01	1.71	1.29	0.76
IDA blend	1961 – 2015	2.21	1.93	4.02	0.36	0.10	0.16	−0.74	2.44	3.77	0.68	1.89
Indonesia	1961 – 2015	3.53	3.57	3.32	0.35	1.88	1.87	1.31	1.67	2.57	2.31	1.29
IDA only	1967 – 2015	2.65	0.92	1.62	0.56	1.03	1.68	0.60	0.57	1.26	1.26	0.40
Isle of Man	1985 – 2013	3.67	5.83	3.61	0.13	3.66	3.47	3.51	1.49	1.71	3.51	1.49
India	1961 – 2015	6.28	3.29	3.10	0.19	2.21	2.58	2.04	1.17	1.45	2.32	1.02
Ireland	1971 – 2015	7.27	3.56	2.71	0.62	1.41	2.93	−0.31	1.82	4.52	2.36	1.19
Iran, Islamic Rep.	1961 – 2014	3.02	1.86	7.10	0.51	−7.19	−6.80	−13.20	9.67	19.20	−3.51	6.98
Iraq	1969 – 2015	−1.12	4.65	16.70	−0.35	−11.80	−10.50	−8.33	21.20	15.60	−23.10	32.60
Iceland	1961 – 2015	2.88	2.65	3.38	0.42	0.99	1.06	0.14	1.91	3.26	1.49	1.46
Israel	1961 – 2015	0.47	2.60	3.09	0.28	1.61	1.22	1.30	1.30	1.79	1.88	1.10
Italy	1961 – 2015	0.74	2.01	2.21	0.48	1.47	1.06	0.95	0.91	1.72	1.81	0.72
Jamaica	1967 – 2015	0.70	0.66	4.21	0.24	−0.52	−0.51	−0.97	2.29	3.00	−0.16	1.92
Jordan	1976 – 2015	−0.02	1.81	5.11	0.27	−0.85	−1.18	−1.68	3.52	4.84	−0.71	3.89
Japan	1961 – 2015	0.61	3.04	2.99	0.52	1.25	0.41	0.07	1.79	3.66	1.99	1.34
Kazakhstan	1991 – 2015	−0.27	3.18	3.46	0.75	−2.76	−4.48	−10.20	4.50	16.10	−1.25	4.77
Kenya	1961 – 2015	2.93	1.68	4.04	0.07	0.70	0.75	0.61	1.75	1.88	0.35	1.95
Kyrgyz Republic	1987 – 2015	1.36	0.56	5.94	0.59	−6.90	−6.59	−13.70	7.98	18.60	−3.25	5.58
Cambodia	1994 – 2015	5.32	5.53	2.43	0.48	3.63	3.56	3.01	1.10	2.08	4.02	0.80
Kiribati	1971 – 2015	1.35	0.07	11.30	0.28	−14.00	−13.70	−18.30	17.30	24.10	−10.90	14.30
St. Kitts and Nevis	1978 – 2015	3.44	3.49	3.58	0.50	0.92	0.90	−0.58	2.47	4.83	1.88	1.76
Korea, Rep.	1961 – 2015	2.22	5.98	3.62	0.26	3.49	2.84	3.11	1.74	2.34	3.72	1.46
Kosovo	2001 – 2015	4.53	3.22	1.44	−0.19	3.01	2.84	3.03	0.18	0.15	0.78	4.10
Kuwait	1966 – 2015	−3.96	−2.71	8.49	0.20	−8.25	−8.41	−9.64	8.82	11.00	−7.17	7.72
Latin America & Caribbean (ehi)	1961 – 2015	−2.20	1.79	2.19	0.41	1.47	0.37	1.13	0.79	1.33	1.73	0.59

Country	data	x_0	μ	σ	k	$\bar{r}^f_{0,1}$	$r^f_{0,1}$	$\bar{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Lao PDR	1985 – 2015	5.22	4.33	2.79	0.28	2.96	3.12	2.70	1.07	1.48	3.10	0.89
Lebanon	1989 – 2015	−2.63	3.29	8.78	0.01	−3.05	−3.08	−3.09	7.78	7.86	−11.10	15.80
Liberia	1961 – 2015	−2.08	−0.78	15.80	0.44	−37.40	−37.80	−57.30	41.10	72.50	−25.80	31.60
Libya	2000 – 2015	−10.50	1.63	27.80	−0.48	−38.80	−34.90	−27.50	55.20	36.90	−86.50	106.00
St. Lucia	1981 – 2015	1.02	2.47	6.49	−0.13	−0.28	−0.16	−0.01	3.76	3.32	−1.04	4.41
Latin America & Caribbean	1961 – 2015	−1.98	1.73	2.10	0.40	1.49	0.50	1.20	0.72	1.19	1.73	0.54
Least developed countries	1985 – 2015	2.00	1.81	1.49	0.77	1.01	1.11	−0.55	0.89	3.32	1.62	0.63
Low income	1983 – 2015	1.91	0.77	1.49	0.74	0.53	1.09	−0.67	0.78	2.65	1.03	0.52
Liechtenstein	1971 – 2009	−1.93	2.47	2.41	0.55	1.39	−0.23	0.47	1.24	2.69	1.87	0.87
Sri Lanka	1962 – 2015	3.81	3.52	2.00	0.35	2.81	2.87	2.60	0.60	0.92	2.92	0.48
Lower middle income	1961 – 2015	3.96	2.44	1.64	0.53	2.07	2.61	1.69	0.56	1.16	2.29	0.39
Low & middle income	1961 – 2015	2.24	2.86	1.51	0.61	2.30	2.05	1.77	0.56	1.40	2.56	0.39
Lesotho	1961 – 2014	2.35	3.24	5.68	0.07	0.35	0.31	0.19	3.45	3.71	0.35	3.32
Late-demographic dividend	1961 – 2015	2.74	4.00	2.10	0.53	2.77	2.32	2.15	0.90	1.87	3.05	0.68
Lithuania	1996 – 2015	2.37	5.52	5.26	0.23	1.64	1.15	0.96	3.56	4.63	2.13	3.08
Luxembourg	1961 – 2015	2.39	2.57	3.41	0.20	1.51	1.49	1.30	1.42	1.77	1.69	1.23
Latvia	1996 – 2015	2.68	5.51	4.87	0.55	−0.50	−1.55	−4.28	5.10	11.00	1.61	3.60
Macao SAR, China	1983 – 2015	−21.70	4.58	7.85	0.46	−6.32	−14.30	−12.00	10.90	19.90	−2.16	7.49
Morocco	1967 – 2015	3.00	2.83	3.23	−0.42	2.33	2.28	2.47	0.75	0.52	1.83	1.35
Monaco	1971 – 2008	7.89	1.95	2.18	0.55	1.28	3.45	0.55	1.01	2.16	1.77	0.68
Moldova	1981 – 2015	−0.44	0.16	8.86	0.29	−8.26	−8.37	−11.00	10.70	15.10	−6.29	8.83
Madagascar	1961 – 2015	0.20	−0.94	3.87	−0.04	−0.77	−0.81	−0.74	1.44	1.39	−0.90	1.53
Maldives	2002 – 2015	−0.51	4.12	6.12	−0.42	1.73	3.03	2.24	2.71	1.89	−0.31	4.86
Middle East & North Africa	1969 – 2015	1.05	1.76	3.20	0.46	0.43	0.21	−0.54	1.83	3.37	1.07	1.43
Mexico	1961 – 2015	1.23	1.79	3.10	0.25	1.11	1.02	0.85	1.26	1.68	1.32	1.05
Marshall Islands	1982 – 2014	−1.18	1.24	5.58	−0.03	−0.57	−0.51	−0.50	3.01	2.91	−0.87	3.21
Middle income	1961 – 2015	2.39	2.97	1.53	0.61	2.35	2.11	1.80	0.58	1.44	2.62	0.40
Macedonia, FYR	1991 – 2015	3.52	1.48	2.43	0.62	0.41	1.25	−0.99	1.46	3.66	0.69	1.32
Mali	1968 – 2015	4.47	2.87	5.53	−0.10	0.71	0.61	0.87	2.81	2.56	0.28	3.15
Malta	1971 – 2013	1.95	4.21	3.24	0.69	−0.02	−1.07	−4.15	3.23	9.69	2.07	2.06

Country	data	x_0	μ	σ	k	$\bar{r}^f_{0,1}$	$r^f_{0,1}$	$\bar{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Myanmar	1961 – 2015	6.08	3.32	5.23	0.36	−0.53	0.13	−1.99	4.16	6.47	0.42	3.27
Middle East&North Africa (ehi)	1966 – 2014	−0.21	1.72	4.42	0.33	−0.38	−0.81	−1.27	2.85	4.26	0.30	2.25
Montenegro	1998 – 2015	3.28	2.20	4.49	0.02	0.82	0.83	0.79	2.06	2.10	0.76	2.18
Mongolia	1982 – 2015	0.59	3.07	3.89	0.67	−1.91	−3.02	−7.00	4.32	12.30	0.72	2.85
Mozambique	1981 – 2015	3.37	3.12	6.36	0.36	−2.44	−2.38	−4.59	6.14	9.53	−0.92	4.79
Mauritania	1961 – 2014	1.66	0.94	5.69	−0.03	−0.88	−0.89	−0.81	3.15	3.05	−1.19	3.54
Mauritius	1977 – 2015	3.36	3.52	3.09	0.12	2.46	2.44	2.36	1.08	1.23	2.54	1.00
Malawi	1961 – 2015	−0.16	1.37	5.02	−0.04	−0.02	0.01	0.03	2.44	2.35	−0.20	2.59
Malaysia	1961 – 2015	3.47	3.83	3.18	0.11	2.63	2.61	2.55	1.12	1.26	2.69	1.04
North America	1961 – 2015	1.49	2.06	1.87	0.31	1.94	1.82	1.80	0.49	0.72	2.03	0.40
Namibia	1981 – 2015	3.26	1.04	2.89	0.43	0.37	1.01	−0.30	1.42	2.47	0.76	1.07
Niger	1961 – 2015	−0.48	−0.72	5.55	0.04	−2.08	−2.07	−2.18	3.22	3.37	−2.08	3.16
Nigeria	1961 – 2015	−0.01	1.61	7.68	0.30	−5.15	−5.48	−7.41	8.24	11.80	−3.50	6.64
Nicaragua	1961 – 2015	3.75	0.47	5.49	0.36	−2.76	−1.98	−4.32	4.53	7.00	−1.59	3.54
Netherlands	1961 – 2015	1.56	2.17	1.89	0.44	1.87	1.69	1.58	0.61	1.07	2.01	0.47
Norway	1961 – 2015	0.45	2.46	1.54	0.57	2.09	1.33	1.67	0.53	1.20	2.36	0.38
Nepal	1961 – 2015	2.13	1.80	2.56	−0.22	1.78	1.73	1.84	0.54	0.45	1.60	0.70
New Zealand	1978 – 2015	1.46	1.43	1.93	0.32	1.48	1.49	1.32	0.54	0.79	1.57	0.43
OECD members	1961 – 2015	1.33	2.19	1.53	0.48	2.05	1.78	1.81	0.43	0.80	2.22	0.31
Oman	1961 – 2015	−2.37	8.01	27.60	0.09	−61.40	−62.00	−66.70	82.90	91.50	−59.30	78.60
Other small states	2001 – 2015	0.60	3.66	2.52	0.22	2.76	2.32	2.62	0.80	1.02	2.78	0.72
Pakistan	1961 – 2015	3.37	2.45	2.19	0.17	2.16	2.26	2.08	0.57	0.69	2.23	0.50
Panama	1961 – 2015	4.12	2.81	3.79	0.44	0.56	0.94	−0.62	2.47	4.33	1.38	1.86
Peru	1961 – 2015	1.93	1.57	4.23	0.43	−0.81	−0.70	−2.25	3.05	5.34	0.19	2.26
Philippines	1961 – 2015	4.17	1.71	2.54	0.52	0.85	1.71	−0.02	1.30	2.66	1.39	0.90
Palau	1992 – 2015	8.37	0.10	5.67	0.22	−2.38	−1.16	−3.10	4.05	5.19	−2.41	3.87
Papua New Guinea	1961 – 2014	6.28	1.57	3.98	0.52	−1.05	0.58	−3.12	3.14	6.39	0.24	2.21
Poland	1991 – 2015	3.68	4.15	1.56	0.28	3.48	3.39	3.39	0.33	0.46	2.75	0.85
Pre-demographic dividend	1969 – 2015	0.22	1.21	4.61	0.12	−0.17	−0.25	−0.38	2.39	2.73	−0.03	2.34
Puerto Rico	1961 – 2013	0.57	2.74	2.52	0.58	1.35	0.51	0.19	1.43	3.26	2.04	0.98

Country	data	x_0	μ	σ	k	$\overline{r}^f_{0,1}$	$r^f_{0,1}$	$\overline{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Portugal	1961 – 2015	1.97	2.96	3.24	0.52	0.91	0.57	−0.44	2.09	4.22	1.77	1.46
Paraguay	1961 – 2015	1.67	2.46	3.72	0.27	1.03	0.88	0.59	1.86	2.55	1.39	1.53
Pacific island small states	1982 – 2015	2.17	0.79	2.21	−0.26	1.23	0.99	1.28	0.40	0.31	0.87	0.65
Post-demographic dividend	1961 – 2015	1.35	2.35	1.54	0.48	2.14	1.82	1.89	0.44	0.83	2.31	0.32
French Polynesia	1966 – 2000	2.11	1.50	4.30	−0.45	1.03	0.84	1.29	1.31	0.89	0.03	2.46
Qatar	2001 – 2015	0.65	1.58	3.25	−0.57	1.54	1.89	1.70	0.70	0.44	0.60	1.69
Romania	1991 – 2015	4.14	3.10	3.92	0.45	0.50	0.82	−0.87	2.71	4.88	0.02	3.17
Russian Federation	1990 – 2015	−3.93	0.83	5.13	0.65	−6.28	−8.36	−14.00	7.04	19.20	−2.58	4.83
Rwanda	1961 – 2015	4.44	2.08	9.51	−0.22	−3.38	−3.72	−2.51	7.53	6.14	−5.89	9.81
South Asia	1961 – 2015	5.78	3.06	2.57	0.25	2.29	2.75	2.11	0.87	1.16	2.43	0.72
Saudi Arabia	1969 – 2015	1.34	1.05	5.39	0.56	−4.73	−4.62	−9.64	6.30	14.00	−1.92	4.37
Sudan	1961 – 2015	1.17	1.42	4.97	0.31	−1.15	−1.21	−2.16	3.52	5.11	−0.46	2.83
Senegal	1961 – 2015	3.27	−0.02	3.29	−0.32	0.36	−0.35	0.48	0.84	0.63	−0.03	1.22
Singapore	1961 – 2015	0.81	5.18	4.11	0.28	2.44	1.62	1.87	2.31	3.21	2.90	1.86
Solomon Islands	1991 – 2015	1.23	0.64	5.17	0.48	−3.26	−3.07	−5.99	4.89	9.21	−1.54	3.64
Sierra Leone	1961 – 2015	−22.00	0.65	6.67	0.03	−2.24	−2.62	−2.31	4.56	4.68	−2.35	4.53
El Salvador	1966 – 2015	2.14	0.99	2.52	0.73	−1.04	−0.48	−4.28	2.17	7.23	0.52	1.42
San Marino	1971 – 2008	0.93	2.54	2.53	0.10	2.11	2.00	2.06	0.71	0.79	2.10	0.67
Somalia	1961 – 1990	−2.05	−0.13	7.59	−0.32	−2.44	−2.03	−1.75	4.46	3.34	−4.83	6.76
Serbia	1996 – 2015	1.18	3.11	4.67	0.02	1.28	1.25	1.25	2.22	2.28	1.14	2.29
Sub-Saharan Africa (ehi)	1961 – 2015	0.24	0.86	2.42	0.49	0.52	0.32	−0.11	1.09	2.09	0.89	0.79
Sub-Saharan Africa	1961 – 2015	0.24	0.86	2.42	0.49	0.52	0.32	−0.11	1.09	2.08	0.90	0.79
Small states	2001 – 2015	0.73	3.34	2.25	0.23	2.67	2.27	2.55	0.65	0.85	2.69	0.58
Sao Tome and Principe	2001 – 2014	2.26	2.87	1.74	−0.21	2.72	2.80	2.74	0.26	0.21	2.52	0.37
Suriname	1976 – 2015	0.62	0.72	4.80	0.27	−1.18	−1.20	−1.90	3.08	4.21	−0.74	2.59
Slovak Republic	1993 – 2015	3.49	4.06	3.04	0.28	2.60	2.50	2.29	1.27	1.76	2.73	1.08
Slovenia	1996 – 2015	2.79	2.33	3.11	0.35	1.25	1.36	0.76	1.45	2.22	1.62	1.17
Sweden	1961 – 2015	3.00	2.00	2.06	0.33	1.78	2.00	1.59	0.62	0.93	1.95	0.51

Country	data	x_0	μ	σ	k	$\bar{r}^f_{0,1}$	$r^f_{0,1}$	$\bar{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Swaziland	1971 – 2015	0.25	2.28	4.00	0.18	0.91	0.67	0.65	1.92	2.33	1.10	1.85
Seychelles	1961 – 2015	1.83	3.27	5.91	0.03	0.30	0.28	0.24	3.58	3.68	−0.09	3.77
Syrian Arab Republic	1961 – 2007	1.91	2.50	7.55	−0.23	−0.95	−0.86	−0.39	4.73	3.83	−2.42	6.19
Chad	1961 – 2015	−1.47	0.91	7.82	0.15	−4.34	−4.59	−5.17	7.14	8.45	−3.73	6.38
East Asia & Pacific (IDA)	1961 – 2015	5.70	5.99	2.39	0.43	4.08	4.00	3.63	0.97	1.69	3.60	1.42
Europe & Central Asia (IDA)	1990 – 2015	−0.57	1.53	3.76	0.64	−2.10	−2.99	−6.02	3.73	9.89	−0.23	2.58
Togo	1961 – 2015	2.74	0.82	5.50	0.13	−1.28	−1.12	−1.59	3.43	3.92	−1.10	3.32
Thailand	1961 – 2015	2.47	4.39	2.99	0.42	2.52	1.97	1.83	1.51	2.60	2.97	1.12
Tajikistan	1986 – 2015	1.91	−0.79	7.22	0.71	−17.70	−16.50	−39.00	15.60	48.90	−8.45	10.70
Turkmenistan	1988 – 2015	5.19	3.67	10.30	0.13	−6.62	−6.49	−7.77	12.20	14.00	−5.92	11.40
Latin America&Caribbean (IDA)	1961 – 2015	−2.02	1.71	2.18	0.40	1.44	0.46	1.13	0.76	1.25	1.68	0.57
Middle East&North Africa (IDA)	1966 – 2014	−0.18	1.75	4.47	0.32	−0.37	−0.78	−1.21	2.87	4.20	0.29	2.28
Timor-Leste	2000 – 2015	1.50	3.00	6.39	0.29	−1.92	−2.21	−3.37	5.63	7.93	−1.46	6.03
Tonga	1982 – 2014	1.71	1.38	2.45	0.13	1.36	1.39	1.29	0.68	0.79	1.44	0.66
South Asia (IDA & IBRD)	1961 – 2015	5.78	3.06	2.57	0.25	2.29	2.75	2.11	0.87	1.16	2.43	0.72
Sub-Saharan Africa (IDA)	1961 – 2015	0.24	0.86	2.42	0.49	0.52	0.32	−0.11	1.09	2.08	0.90	0.79
Trinidad and Tobago	1961 – 2015	0.59	2.13	3.74	0.59	−1.00	−1.61	−3.82	3.26	7.68	0.49	2.45
Tunisia	1966 – 2015	−0.21	2.83	3.33	−0.01	2.01	2.04	2.02	1.10	1.08	1.92	1.14
Turkey	1961 – 2015	2.48	2.60	3.74	−0.04	1.66	1.67	1.70	1.35	1.29	1.47	1.46
Tuvalu	1991 – 2014	1.81	1.85	5.48	0.01	−0.20	−0.20	−0.22	3.03	3.06	−0.36	3.15
Tanzania	1989 – 2015	3.68	2.24	1.69	0.61	1.74	2.33	1.09	0.70	1.72	2.05	0.48
Uganda	1983 – 2015	1.68	2.48	2.58	0.49	1.45	1.19	0.72	1.25	2.39	1.91	0.90
Ukraine	1988 – 2015	−9.57	−0.98	6.44	0.68	−13.00	−16.90	−27.40	11.70	34.40	−6.14	7.87
Upper middle income	1961 – 2015	2.18	3.36	1.77	0.57	2.51	2.06	1.95	0.70	1.59	2.80	0.49
Uruguay	1961 – 2015	0.63	1.82	3.62	0.52	−0.35	−0.75	−2.03	2.59	5.24	0.69	1.82
United States	1961 – 2015	1.63	2.08	1.89	0.31	1.94	1.85	1.80	0.51	0.73	2.03	0.41
Uzbekistan	1988 – 2015	6.13	2.45	3.07	0.77	−2.49	−0.60	−9.26	3.78	14.30	0.68	2.47
St. Vincent and Grenadines	1961 – 2015	1.36	2.58	5.86	−0.01	0.01	0.02	0.04	3.39	3.35	−0.19	3.50
Venezuela, RB	1961 – 2015	−6.96	0.19	5.01	0.19	−1.45	−2.38	−1.92	3.06	3.80	−1.09	2.65

Country	data	x_0	μ	σ	k	$\overline{r}^f_{0,1}$	$r^f_{0,1}$	$\overline{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$	r^{f*}	π^*
Vietnam	1985 – 2015	5.55	4.96	1.04	0.73	3.80	4.09	3.21	0.39	1.32	3.98	0.31
Vanuatu	1980 – 2014	0.04	1.09	3.84	0.25	0.05	−0.13	−0.36	1.94	2.60	−0.47	2.30
West Bank and Gaza	1995 – 2015	9.15	3.16	9.72	0.21	−6.89	−6.04	−8.90	11.80	15.00	−5.57	10.40
World	1961 – 2015	1.27	1.85	1.35	0.37	1.98	1.83	1.88	0.28	0.45	2.07	0.22
Samoa	1983 – 2015	0.91	1.57	3.16	0.22	0.97	0.88	0.75	1.26	1.62	1.10	1.09
Yemen, Rep.	1991 – 2013	1.50	0.39	3.91	0.05	−0.04	0.00	−0.09	1.60	1.68	−0.06	1.60
South Africa	1961 – 2015	−0.37	1.02	2.13	0.51	0.80	0.33	0.24	0.89	1.78	1.15	0.63
Congo, Dem. Rep.	1961 – 2015	3.61	−1.31	5.24	0.40	−3.91	−2.59	−5.74	4.41	7.30	−3.04	3.63
Zambia	1961 – 2015	−1.42	0.28	4.61	0.12	−0.78	−0.92	−0.99	2.39	2.71	−0.67	2.20
Zimbabwe	1961 – 2015	−1.24	0.02	6.08	0.40	−4.40	−4.73	−6.82	5.91	9.75	−2.69	4.49

Appendix C: Term structures for a subset of 10 countries and economic zones

For the following 10 figures, the plain curves correspond to the term structures of either interest rates (left) or aggregate risk premia (right) using the model described in Section 2. The dotted curves corresponds to the equilibrium interest rates and aggregate risk premia when assuming that shocks are temporary ($k = 1$) and parameters (μ, σ) are estimated accordingly. We assume that $\gamma = 10$, $\rho = 2/3$ and $\delta = 1\%$.

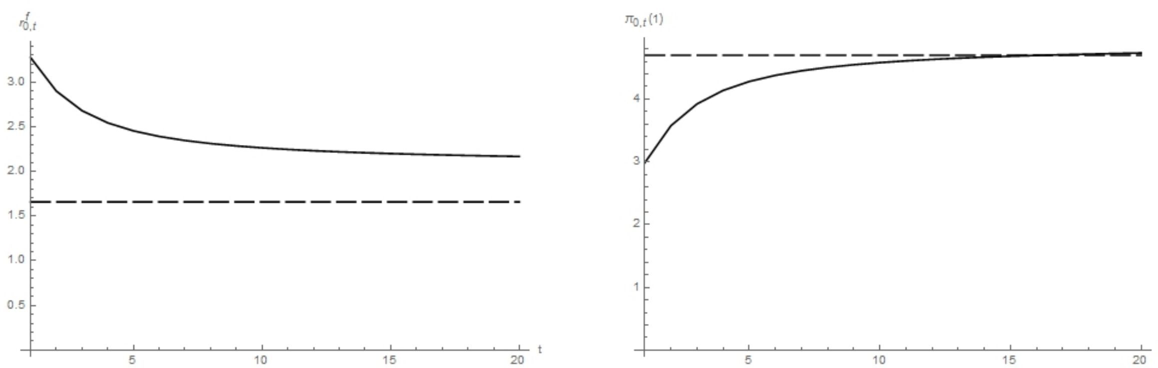

Figure 1: China

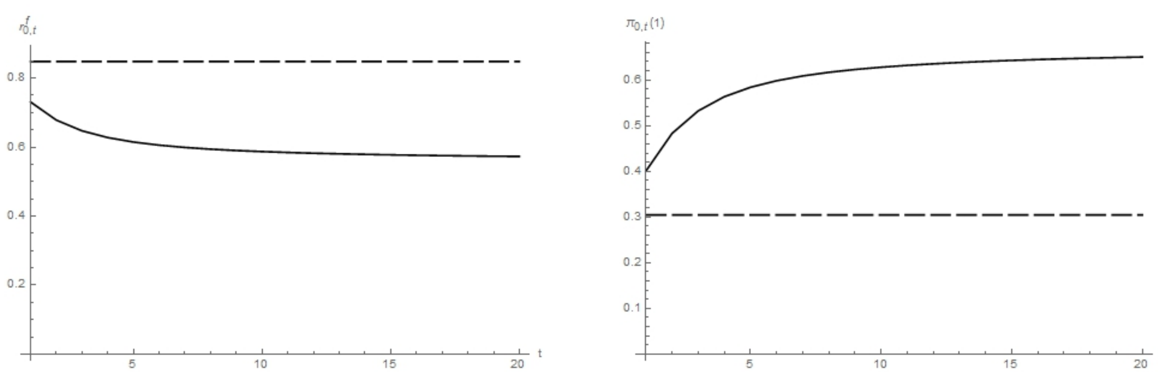

Figure 2: European Union

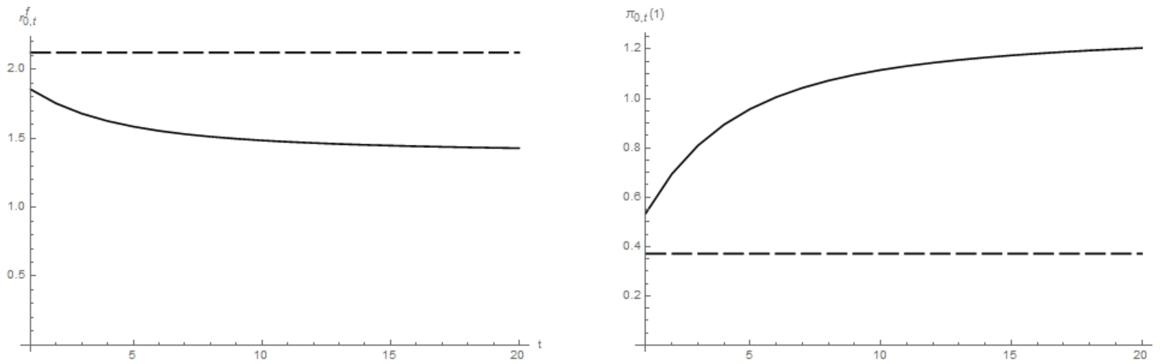

Figure 3: France

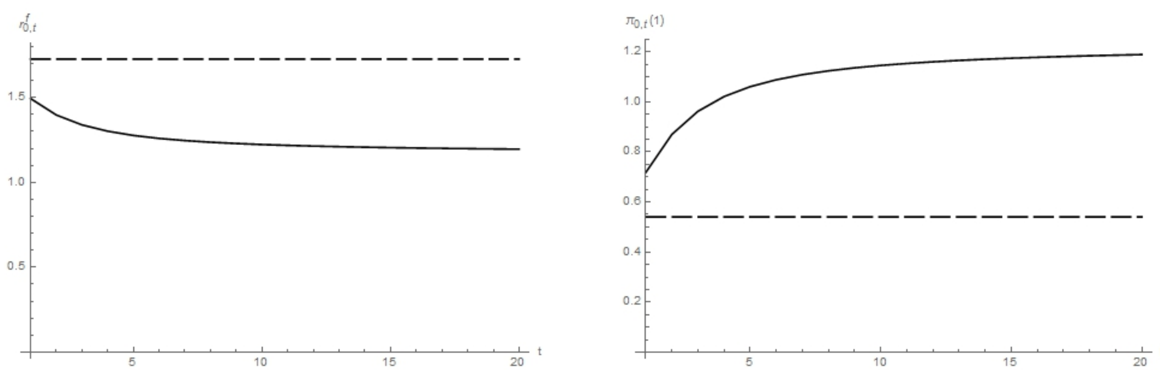

Figure 4: Latin America and Caribbean

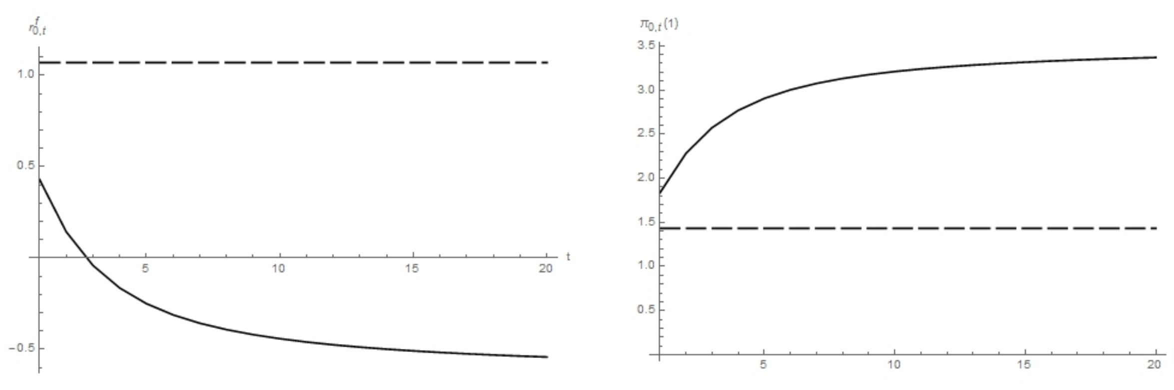

Figure 5: Middle East and North Africa

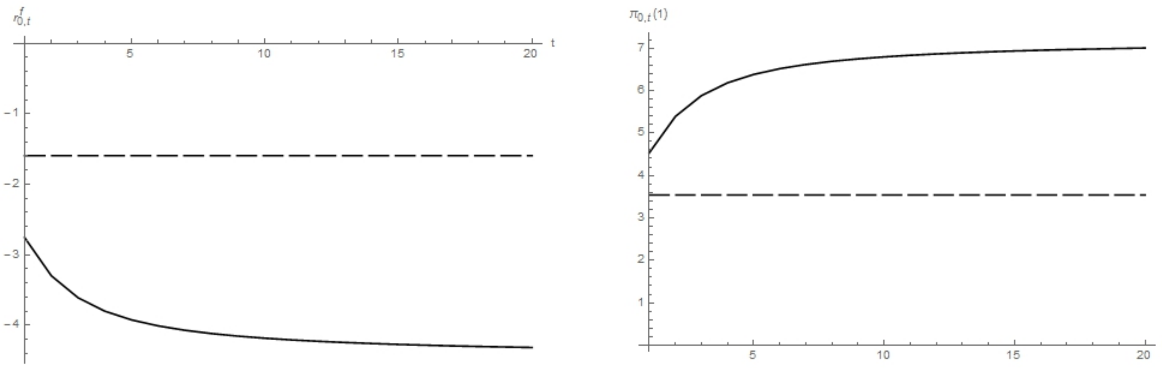

Figure 6: Nicaragua

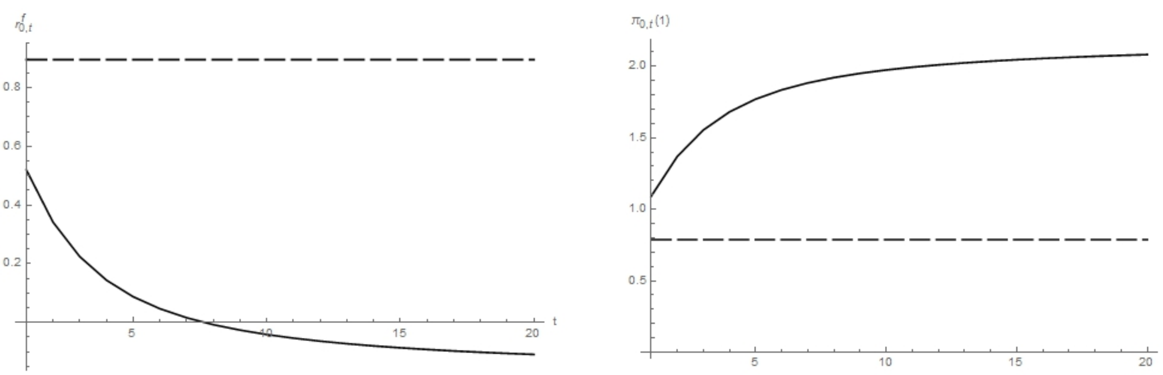

Figure 7: Sub-Saharan Africa

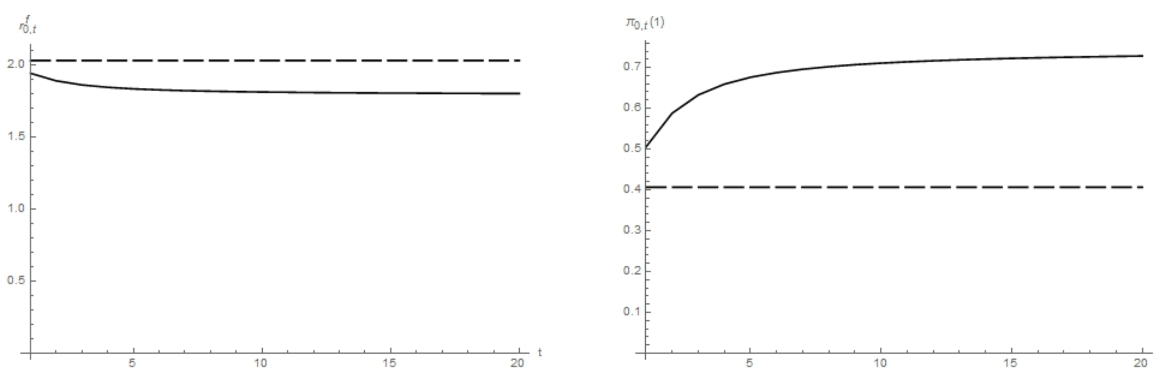

Figure 8: United States

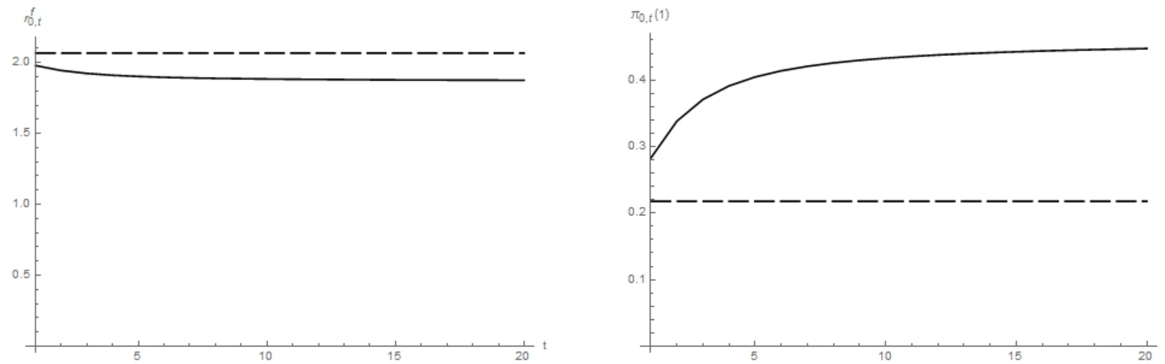

Figure 9: World

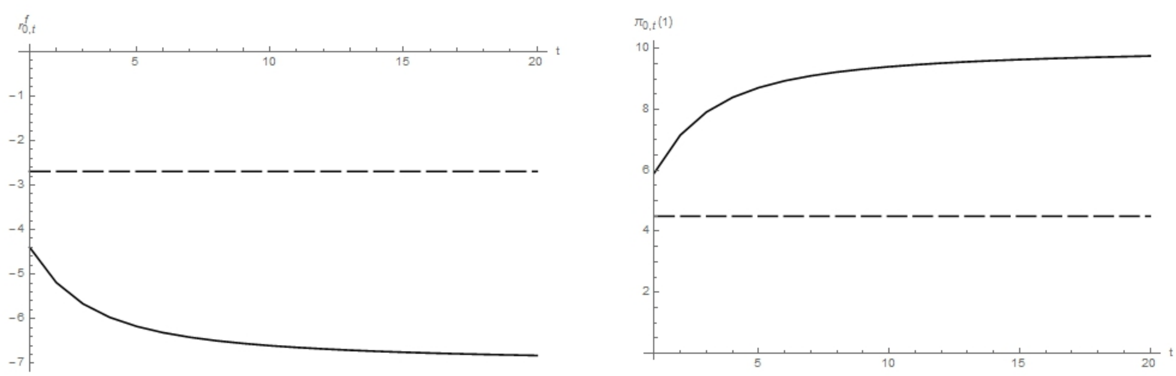

Figure 10: Zimbabwe

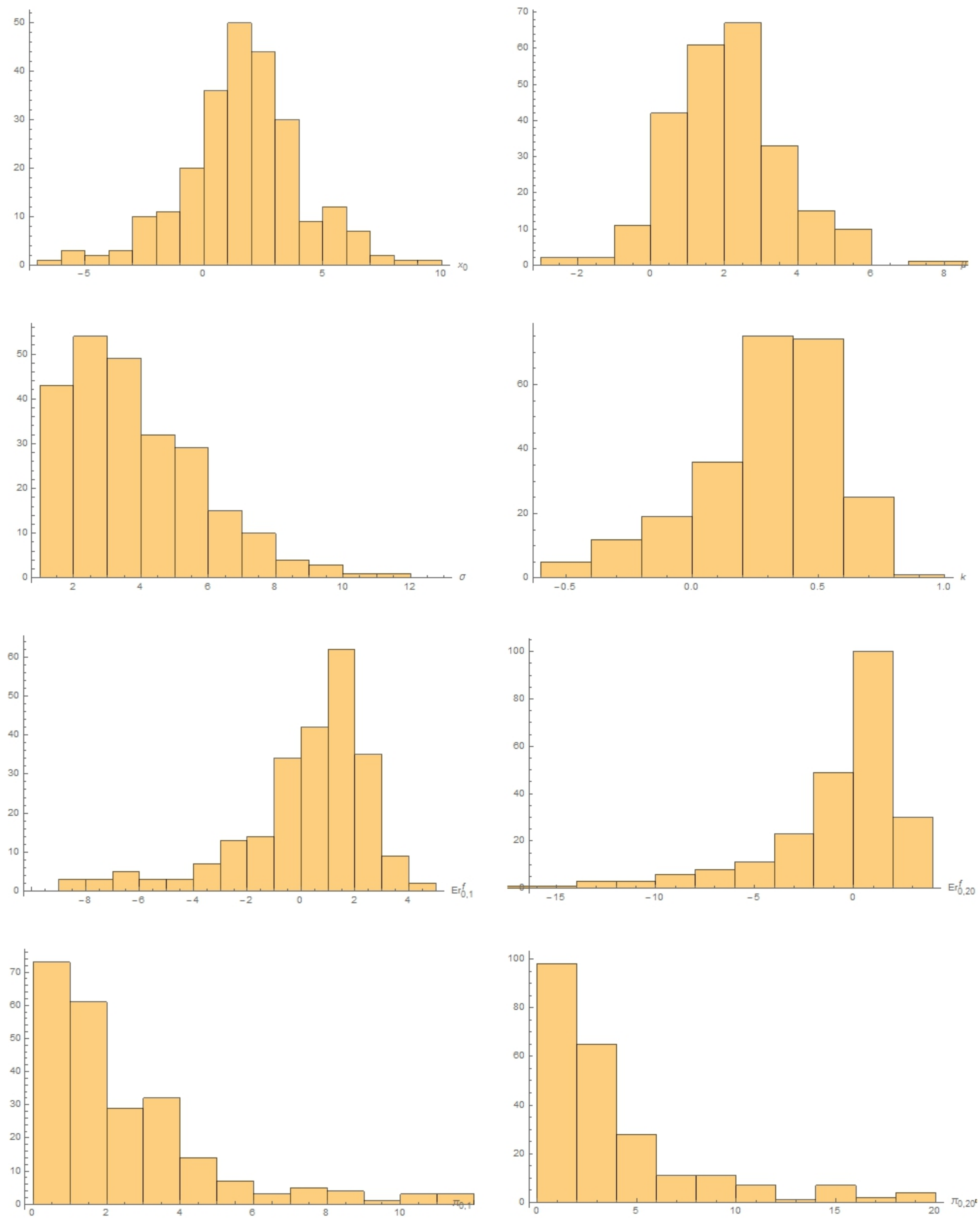

Figure 11: Histograms for $(x_0, \mu, \sigma, k, \overline{r}^f_{0,1}, \overline{r}^f_{0,20}, \pi_{0,1}, \pi_{0,20})$ for the 248 countries and economic zones of the World Bank data base.