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Assessing the Effects of Natural Resources on Long-Term Growth

An Extension of the World Bank Long Term Growth Model

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Abstract

This paper extends the World Bank's Long-Term Growth Model (LTGM) with the addition of a natural resource sector to analyze how long-run growth evolves in resource-rich countries and the growth impacts of price shocks and resource discoveries. In the LTGM-Natural Resource Extension (LTGM-NR), commodity price shocks affect long-term economic growth through physical investment rates. As a large share of resource income typically accrues to the government, the size of the boost to investment in a price boom depends on the government's fiscal rule. Fiscal rules that prioritize public investment, like a Hartwick Rule, generally lead to the largest increases in long-term growth. However, structural surplus rules, which save commodity revenues, can also boost growth if they free up savings for private investment. The response of growth to discoveries of natural resources is similar to the response to price shocks, although discoveries also produce a direct effect on real GDP, in addition to an indirect effect through investment. The LTGM-NR also captures the effect of other (non-resource) growth fundamentals in resource-rich economies, and it is better suited to general growth analysis in these countries than the standard LTGM. However, the LTGM-NR is a supply-side model, and so does not capture the short-run effects of price and discovery shocks that operate through aggregate demand.

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Assessing the Effects of Natural Resources on Long-Term Growth:
An Extension of the World Bank Long Term Growth Model

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[1] The views expressed here are the authors' and do not necessarily reflect those of the World Bank, its executive directors, or the countries they represent.
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I. Introduction

The celebrated Solow-Swan neoclassical growth model analyzes how long-run economic growth depends on growth fundamentals like productivity, savings, investment, human, and physical capital and demographic trends (Solow 1956; Swan 1956; Mankiw et al. 1992; Hall and Jones 1999). The World Bank's Long Term Growth Model (LTGM; Loayza and Pennings 2018) is in this tradition, though it is applied to simulations of future growth in developing countries. However, standard neoclassical models are inappropriate for economies where the natural resource sector is sufficiently large to have a first-order effect on growth, including many developing countries. Traditional neoclassical models are also silent on how commodity price shocks and resource discoveries affect long-term growth for commodity exporters, and the economic consequences of government policies that manage resource wealth.

The Long Term Growth Model–Natural Resource Extension (LTGM-NR) seeks to fill this gap by augmenting an otherwise-standard neoclassical growth model with a natural resource sector and government fiscal policy. The model is designed to be accessible and transparent—a spreadsheet-based toolkit (without macros) is freely downloadable at https://www.worldbank.org/LTGM with preloaded data for 56 resource-rich countries (see Appendix Table 1 for a list of available countries). The LTGM-NR first allows for the evaluation of how commodity price shocks and discoveries of natural resources affect a country's medium to long-term economic growth and how it depends on different fiscal frameworks. Second, the model analyzes how standard growth fundamentals affect growth in resource-rich economies, like human capital, demographics, and productivity. The LTGM-NR allows for a more accurate analysis of the effect of these fundaments than one-sector models as those models do not account for heterogeneity across sectors or the consequences of depleting reserves of natural resources. However, as the LTGM-NR is a supply-side model, it does not capture the short-run effects of price and discovery shocks that operate through aggregate demand.

The first step in analyzing country-specific commodity price shocks or discoveries is a resource-accounting exercise that evaluates the size of resource exports and reserves in each country, and hence the scale of the direct impact of a given change in commodity prices or a resource discovery. We make this easier by providing preloaded data on the resource sector in 56 countries. Our simulations incorporate some often-misunderstood accounting identities, such as the fact that commodity price fluctuations only directly affect gross domestic income (GDI), whereas real gross domestic product (GDP) changes only indirectly (as real GDP fixes the price of exports, see Kehoe and Ruhl 2008).

The second step is to trace out how higher GDI and government resource revenues affect investment rates and long-term growth. As in a standard neoclassical model, a higher income boosts private savings (assumed to be a fixed share of GDI) and, consequentially, private investment. But more important—since a large share of the resource income typically accrues to the government—is how the government's fiscal policy affects investment.[2] In short, fiscal rules that generate the largest increases in investment will generate the fastest growth in the medium and long terms.

The LTGM-NR has two submodels: the *LTGM-NR-Default*, in which public investment responds directly to fiscal policy via a simple fiscal rule, and the *LTGM-NR-External-Balance* that considers more sophisticated fiscal rules and the relationship between public savings and the international capital flows.

The fiscal rules in both submodels are classified by whether the government saves or spends any extra resource revenues; and if it spends them, whether that spending falls on public investment or government consumption. For a temporary commodity price shock, a *Balanced Budget Rule (BBR)* is when the government spends the extra revenues and a *Structural Surplus Rule (SSR)* when they are mostly saved in financial assets. We usually assume that the spending allocation across investment/government consumption is kept constant, but if all the extra spending is on public investment, we call it a *Hartwick Rule (HR).* This yields four rules with

[2] Cross-country evidence suggests that governments retain on average 65-85 percent of rents in the hydrocarbons sector and 40-60 percent of rents in the mining sector (IMF 2012).

different combinations of spending/savings and spending allocations: BBR, BBR-HR, SSR, and SSR-HR (see Table 1).[3]

Table 1: Taxonomy of fiscal rules for Government resource revenues

		Spending allocation	
		Mostly Govt. consumption (Historical budget shares)	**Public investment** (Hartwick rule)
Timing of spending	**Spent on impact** (Balanced budget rule)	BBR	BBR-HR (“HR” in Default submodel)
	Save for the future (Structural surplus rule)	SSR	SSR-HR (Not in Default submodel)

BBR=Balanced Budget Rule; SSR=Structural Surplus Rule; HR=Hartwick Rule

The LTGM-NR-External-Balance also analyzes how resource revenues and different fiscal rules interact with international capital flows. This mostly affects the SSR, which often leads to large movements in international borrowing/lending. Most important is the effect on the current account balance and private investment. If the current account and private savings are relatively fixed, the extra public savings through an SSR can crowd in private investment dollar-for-dollar, leading the SSR to have a similar path for investment and growth as a BBR-Hartwick rule. In contrast, without crowding-in of investment (full adjustment of the current account), the SSR in the External-Balance model performs similarly to that of the simple model (which abstracts from international flows). Of course, real-world countries are somewhere in between these two extremes, and so the strength of crowding in is chosen via a parameter that is calibrated to match the data. In addition, the reduced international borrowing generated by a fiscal surplus under an SSR can reduce the government’s interest bill–through reduced borrow and lower interest rates–which can free up funds for public investment in the long-run under the SSRs.

Angola Case Study. To illustrate how the LTGM-NR works, we discuss three simulations with the model calibrated to Angola. First, we simulate the baseline

[3] The difference between the SSR and SSR-HR depends on the timing of the analysis. In the short term, revenues are mostly saved, and, consequentially, both rules yield a similar allocation of spending. However, they diverge substantially in the medium and long terms, as fiscal surpluses under the SSRs can improve the government net asset position, freeing up funds for extra spending. In this case, the SSR-HR would lead a higher path of public investment than the SSR.

"business-as-usual" growth path for Angola over the next three decades. We find that, in the absence of major economic shocks or reforms, potential GDP per capita growth declines slightly in the medium term but is expected to pick up in the longer term. A growth decomposition shows that these "U-shaped" dynamics is driven by the interaction of improving demographic trends, depleting oil reserves, and a transition of the economy away from oil. Finally, we show that an unadjusted ("naïve") calibration of the standard LTGM would generate an overly optimistic growth path for Angola. This is because a one-sector model cannot account for the depletion of oil reserves that is a drag on growth and the fact that the oil sector is much more capital intensive than the non-oil sector.

Second, we evaluate the effects of a hypothetical oil-price boom-and-bust cycle in Angola under each fiscal rule. At the end of the oil price simulation (when oil prices are back at their original level), GDP per capita is highest under the BBR-HR at 20%-25% above the baseline, as all of the higher government oil revenues are invested during the boom years. However, under the other fiscal rules, GDP is only 5%-12% above baseline because of a much smaller increase in investment. The SSR-HR is the only rule that supports faster growth for several years after the end of the oil price cycle. This is because the interest from the extra savings during the oil-price cycle is recycled into the budget, releasing resources for an almost permanent increase in public investment. Third, we find that discoveries have a large and persistent impact on Angola's growth rate, especially under HRs. As before, the LTGM-NR only simulates the supply-side effects of these price shocks and discoveries, not their short effects through aggregate demand.

Related literature While there is a rich literature on managing the short-term cyclical effects of commodity booms (for example, Kumhof and Laxton 2013, Mendes and Pennings 2020, Pieschacón 2012) and whether natural resources are a curse or a blessing for development (see Van der Ploeg 2011 for a survey), the mechanics of medium-to-long-term growth in individual resource-rich economies are much less frequently studied.[4] The closest work is the modeling sections of

[4] For empirical evidence on the resource curse in developing countries, see Terry Lynn (1999) and Wood (1999). For individual country experience with the resource curse (Ghana and Angola), see

Hansen and Gross (2018) and Arezki et al. (2018), who evaluate the effect of exploration and discoveries on medium-term macroeconomic aggregates. While these models share some similarities with ours (in particular, the setup of the resource sector), their purposes are very different. Those papers seek to explain estimated empirical relationships, whereas we propose a simple and accessible tool for country-specific growth simulations and policy analysis. Our model is simpler but is calibrated to 56 countries individually, rather than to one representative small open economy.[5]

The remainder of the paper is organized as follows. Section II describes the theoretical underpinnings of the LTGM-NR, and Section III discusses a general calibration. Section IV presents the applications to Angola, and Section V concludes.

II. Model Description

The point of departure of the LTGM-NR from a standard neoclassical model (like the standard LTGM), is the disaggregation of the economy into non-resource sector (Sector 0) and resource sector (Sector R).

The non-resource sector. The structure of the non-resource sector is identical to the standard LTGM. A representative firm employs physical capital and effective labor with a constant returns to scale Cobb-Douglas production function to generate non-resource output, Y_t^0,

$$Y_t^0 = A_t^0 (h_t L_t)^{\beta} (K_{t-1}^0)^{1-\beta}, \quad 0 < \beta < 1 \tag{1}$$

where A_t^0 is the total factor productivity (TFP) in sector 0, K_{t-1}^0 is the physical capital in sector 0 at the end of period $t-1$, and β is the labor share in the non-resource sector. Effective labor, $h_t L_t$, is decomposed into h_t, human capital per worker, and L_t, the labor force (number of workers). The labor force is defined as $L_t = \varrho_t \omega_t N_t$. Where N_t is total population, ω_t is the working-age to population

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Cust and Mihalyi (2017) for Ghana and Richmond et al. (2013) for Angola. For countries that avoided the resource curve (Chile and Botswana), see Medina and Soto (2007) and AfDB (2016).

[5] Importantly, our model lacks forward-looking decision-making by agents, as this is difficult to incorporate in a spreadsheet-based model.

ratio, and ϱ_t is the labor force participation rate (labor force to working-age population ratio). The variables $A_t^0, h_t, N_t, \omega_t,$ and ϱ_t are exogenous and evolve at the following annual growth rates: $\bar{g}_t^{A_0}, \bar{g}_t^h, \bar{g}_t^N, \bar{g}_t^\omega, \bar{g}_t^\varrho$, respectively. Throughout the model, exogenous variables are indicated with a bar index notation (as in $\bar{g}$).

The natural resource sector. The setup of the natural resource sector builds on Hansen and Gross (2018) and Arezki et al. (2017) in being a Cobb-Douglas function of proven reserves (R) and physical capital (K), with decreasing returns in both R and K (equation (2)).[6] This production function has the desired property that the first reserves are relatively easy to extract – for example, being close to the surface—but later reserves require more and more capital (or technology) to generate the same output as firms are forced to drill further underground or in less accessible locations.[7]

As countries produce multiple commodities, the natural resource sector, R, is further disaggregated into N non-renewable resource industries $i \in \{1, \ldots, N\}$ (e.g., oil, natural gas, copper, gold, and others). As shown in equation (2), the output of resource industry i, Q_t^i, is produced using reserves R_{t-1}^i and physical capital K_{t-1}^i in that industry,

$$Q_t^i = A_t^i \left(K_{t-1}^i\right)^{1-\gamma_i} \left(R_{t-1}^i\right)^{\gamma_i}, \quad 0 < \gamma_i < 1 \text{ and } i \in \{1, \ldots, N\} \tag{2}$$

where A_t^i is the TFP in industry i—which grows at exogenous rate $\bar{g}_t^{A_i}$— and γ_i is the share of resource rents in industry i (then, $1-\gamma_i$ is the capital income share). Capital and reserves are state variables determined in the previous year $t-1$.

The dynamics of reserves in each industry obey the following law of motion,

$$R_t^i = R_{t-1}^i - Q_t^i + \bar{D}_t^i, \quad i \in \{1, \ldots, \mathrm{N}\} \tag{3}$$

[6] Hansen and Gross (2018) and Arezki et al. (2017) also include labor in the production function, but its share is very small (0.13); so for simplicity we exclude it.

[7] Alternatively, R_t^i can be interpreted as a quality-adjusted index of reserves, that takes into consideration geological factors such as ore grade (for minerals) or the composition of hydrocarbons (for petroleum and natural gas). As the highest quality mines and oil fields tend to be explored first, further extraction and depletion reduces the quality of the remaining reserves, scaling down the industry marginal product of capital (see Cochilco 2017). However, our default calibration of the model is not adjusted for the quality of reserves.

where reserves in industry i at the end of period t, R_t^i, increases with an exogenous stream of discoveries, $\overline{D}_t^i$, and is endogenously depleted by the production of good i, Q_t^i.

Equations (4) and (5) describe the evolution of physical capital in activity $j \in \{0, 1, \dots, N\}$ (non-resource sector plus resource industries) and at the aggregate level, respectively,

$$K_t^j = (1-\delta)K_{t-1}^j + I_t^j, \quad j \in \{0, 1, \dots, N\} \tag{4}$$

$$K_t = \sum_{j=0}^{N} K_t^j \tag{5}$$

where δ is the annual depreciation rate (common across all activities), I_t^j is the investment in activity j, and K_t is the aggregate capital at the end of period t.

National income/output and prices. The model economy represents a small, price taking, commodity exporter. The non-resource good is freely traded with a constant price of \$1 (the numeraire), and is used for private and government consumption, investment, and imports. All the proceeds from the resource sector are exported at exogenous international prices $\bar{p}_t^i$ in constant dollars.

Real GDP and real GDI. There are two measures of the "size" of an open economy, Real Gross Domestic Product (RGDP) and Real Gross Domestic Income (RGDI) (also known in the US as "Command-basis GDP").[8] While these two are identical in a closed economy, they are often very different in countries with volatile terms of trade like commodity exporters. The key difference between the measures is how exports are deflated (Kehoe and Ruhl 2008).[9]

For RGDP, exports are deflated by the export price index, which for a country exporting one commodity is simply p_t^1/p_0^1 (the current commodity price relative to its price in the base year, $t = 0$). *This means that changes in commodity prices have*

[8] The term "Real Gross Domestic Income" from the international System of National Accounts (SNA), see https://stats.oecd.org/glossary/detail.asp?ID=2244. The nomenclature can be confusing as the US Bureau of Economic Analysis (BEA) departs from the SNA. The BEA calls GDP(I) (gross domestic product calculated using the income method) GDI, see https://www.bea.gov/resources/learning-center/what-to-know-income-saving, which is why it need to use the term "Command-basis GDI": see https://www.bea.gov/help/glossary/command-basis-gross-domestic-product

[9] RGDP and RGDI are equivalent in a closed economy.

no direct effect on $RGDP_t$ (equation (6)). This is unsurprising, as RGDP is designed to be a measure of *quantities*, which are not directly affected by commodity price shocks.

$$RGDP_t = Y_t^0 + \sum_{i=1}^{N} \bar{p}_t^i Q_t^i / (\bar{p}_t^i / \bar{p}_0^i) = Y_t^0 + \sum_{i=1}^{N} \bar{p}_0^i Q_t^i \tag{6}$$

In contrast, RGDI is a measure of purchasing power: how much can be bought with the national income. Hence, for GDI, exports are deflated by the consumption (or, equivalently, import) price index. In our model, consumption goods are of the non-resource (numeraire) good, and so have a constant price of 1.[10] Hence RGDI, denoted by Y_t (without a superscript), is just the value of non-resource and resource production (in terms of the numeraire)—as in equation (7),

$$Y_t = Y_t^0 + Y_t^R = Y_t^0 + \sum_{i=1}^{N} \bar{p}_t^i Q_t^i \tag{7}$$

While neither RGDP nor RGDI is the "right" measure, we focus more on RGDI. Also, it makes more sense to measure investment and savings relative to RGDI because consumption/investment goods are the numeraire. In this case, let us denote lower case letters variables *as a share of real GDI* (e.g., $z_t \equiv Z_t / Y_t$).

Investment. As in the standard LTGM, capital accumulation is the main endogenous driver of growth in the LTGM-NR. To analyze the effects of different fiscal frameworks on the dynamics of growth in resource-rich countries, we decompose aggregate investment (i_t) into private (i_t^p) and public (i_t^g) investment (see equation (8)), though these are perfect substitutes in the production of new capital.[11] Also, total investment is allocated across sectors and industries of the economy (equation (9)),

$$i_t = i_t^p + i_t^g \tag{8}$$

$$i_t = \sum_{j=0}^{N} i_t^j \tag{9}$$

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[10] Commodity exporting economies typically import a wide range of imported goods (often manufactures), and so the assumption of a constant import price is not too unrealistic.

[11] For a variant of the LTGM where public and private investment are differentiated, see Devadas and Pennings (2019).

In the LTGM-NR, private investment is an exogenous share of GDI (usually a fixed share), and public investment depends on the government's fiscal rule. The assumption of a fixed share of private investment is a generalization of the Solow-Swan tradition.[12] This generalization is an important departure from the literature (Hansen and Gross 2018 and Arezki et al. 2017), where agents are forward looking and allocate investment intertemporally based on its costs and benefits. However, the assumption of a fixed share of private investment keeps the LTGM-NR simple and its mechanisms straightforward. The public investment assumption stems from our application to resource-rich economies, where public investment is often funded by commodity revenues. The determinants of investment vary across the LTGM-NR Default and External-Balance submodels, which are discussed further below.

We also need to determine the allocation of aggregate investment across the non-resource and different resource industries. Again, to keep the model simple, this is done via a rule of thumb where investment is allocated across the different activities proportionally to (i) the marginal efficiency of capital and (ii) the sector's relative size (in terms of capital shares), as below:

$$\frac{i_t^i}{i_t} = \left(\frac{K_{t-1}^i}{K_{t-1}}\right)\left(\frac{MRPK_t^i}{MRPK_t^{DS}}\right)^{\mu}, \quad for\ i \in \{1, \dots, N\} \tag{10}$$

$$MRPK_t^j = (1-\gamma_j)\,\bar{p}_t^j Q_t^j / K_{t-1}^j \quad for\ j \in \{0, 1, \dots, N\} \tag{11}$$

$$MRPK_t^{DS} = \left[\sum_{j=0}^{N}\left(\frac{K_{t-1}^j}{K_{t-1}}\right)\left(MRPK_t^j\right)^{\mu}\right]^{1/\mu} \tag{12}$$

where $MRPK_t^j$ denotes the marginal revenue product of capital (the dollar value of the marginal product of capital) in activity j, $1-\gamma_0 \equiv 1-\beta$, and $MRPK_t^{DS}$ is a

[12] More specifically, in the closed economy Solow-Swan model (without a government), savings are fixed as a share of GDP, which means that investment is also a fixed share of GDP.

Dixit-Stiglitz (DS) aggregator of the MRPK across all activities.[13] The aggregator weights each activity by their capital shares, K_{t-1}^{j}/K_{t-1}.

While the rule of thumb is not derived from an optimizing framework, it is constructed to allocate investment to more efficient and larger industries (as would be the case in an optimizing framework) and has two other appealing properties. First, across same-size industries, if activity i is 1% more efficient than activity j, it receives μ% more investment (i.e., if $K_{t-1}^{i} = K_{t-1}^{j} \rightarrow \ln\left(i_t^i/i_t^j\right) = \mu \ln\left(MRPK_t^i/MRPK_t^j\right)$). Second, investment is allocated so that capital shares remain constant across sectors with the same marginal efficiency of capital (i.e., if $MRPK_t^i = MRPK_t^j \rightarrow K_t^i/K_t^j = K_{t-1}^i/K_{t-1}^j$). Moreover, this rule of thumb is simple enough to be solved in a spreadsheet. We usually calibrate $\mu = 1$.

A. The LTGM-NR Default

In this subsection, we describe the LTGM-NR-Default submodel which is a simplified version of the LTGM-NR-External-Balance (presented in subsection B) and designed to be more user-friendly. In the LTGM-NR-Default, investment is determined directly—like Model 1 in the standard LTGM—rather than indirectly via international and domestic savings. The simplifications fall into two categories, which are related to the determinants of private and public investment, respectively.

First, private investment is assumed to be an exogenous fraction of GDI, which is our analogue of the standard assumption in a Solow-Swan model. Second, public investment as a share of GDI responds to fluctuations in resource revenues according to the following rule:

$$i_t^g = \bar{\imath}_t^g + \theta\left(z_t^R - \breve{z}_t^R\right) + \varepsilon_t \tag{13}$$

where $z_t^R - \breve{z}_t^R$ are cyclical government resource revenues (as a share GDI)—i.e., the deviation of actual resource revenues z_t^R from their "structural" or long-run values $\breve{z}_t^R$ (discussed below). The parameter θ is the marginal propensity to invest

[13] Note that equation (10) does not apply for the non-resource sector (0) but equations (11) and (12) do. Investment in the non-resource sector is determined residually as $I_t^0 = I_t - \sum_{i=1}^{N} I_t^i$. However, the normalization of equation (10) by $MRPK_t^{DS}$ ensures that it also holds for the non-resource sector.

resource revenues: the fraction of cyclical resource revenues that is invested each period.[14] The variable $\bar{\iota}_t^g$ is the exogenous baseline public investment—i.e., public investment as a share of GDI that prevails in the absence of shocks (zero cyclical revenues). The term $\varepsilon_t \equiv -\bar{\iota}_t^g (z_t^R - \breve{z}_t^R)/\tau_R$ is a technical adjustment to prevent double counting, as increases in commodity production will raise real GDI, and hence i_t^g through the constant term $\bar{\iota}^g$. ε_t is usually quantitatively small.

Government resource revenues (as a share of GDI), z_t^R, is obtained from a flat tax rate τ_R applied to resource GDI, y_t^R,[15]

$$z_t^R = \tau_R y_t^R \tag{14}$$

The structural revenue, $\breve{z}_t^R$, is based on structural prices, $\breve{p}_t^i$, and output, $\breve{Q}_t^i$:

$$\breve{z}_t^R = \tau_R \sum_{i=1}^{N} \breve{p}_t^i \breve{Q}_t^i \tag{15}$$

The purpose of defining a structural revenue is to smooth out transitory fluctuations in actual revenues. As discussed in section III, structural prices are usually set at their (perceived) long-term levels. When production is not the focus of the analysis, to keep the model simple, we set structural production equal to actual production or a moving average. However, the LTGM-NR toolkit provides other specifications, such as using baseline as reference production.[16]

Fiscal rules. The marginal propensity to invest, θ, captures the pro-cyclicality of fiscal policy. The user can choose any value of θ, although more common values range between 0 and 1. This range nests three popular fiscal rules:

- $\theta = 0$ captures a SSR (Structural Surplus Rule), as cyclical resource revenues are saved (when prices are high) and do not affect public investment.[17]

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[14] The LTGM-NR spreadsheet also allows the parameter θ to vary over time.
[15] Capturing, for example, royalties from the concessions of exploration of natural resources, tax-receipts from private extractive enterprises and profits from state-owned companies.
[16] An equilibrium is defined as a collection of 15 endogenous trajectories $\{Y_t^0, Q_t^i, R_t^i, K_t^j, K_t, RGDP_t, Y_t, i_t, i_t^0, i_t^i, MRPK_t^j, MRPK_t^{DS}\}$ $\{i_t^g, z_t^R, \breve{z}_t^R\}$ where $j \in \{0,1,\dots,N\}$ with each endogenous variable specified as a function of the exogenous paths $\{\bar{g}_t^{Aj}, \bar{g}_t^h, \bar{g}_t^N, \bar{g}_t^\omega, \bar{g}_t^\varrho, \bar{D}_t^i, \bar{p}_t^i, \bar{\iota}_t^p, \bar{\iota}_t^g, \breve{p}_t^i, \breve{Q}_t^i\}$ and initial conditions $\{GDP_0, GDP_0^j, K_0, K_0^j, R_0^i, p_0^i, N_0, \omega_0, \varrho_0\}$ that satisfy equations (1)—(15) for all *t*.
[17] In the LTGM-NR-default the SSR and SSR-HR are very similar (given that spending is mostly constant). Hence, we focus on the SSR in the applications of the Default submodel.

- $\theta = \theta^{hist}$ captures a BBR (Balanced Budget Rule), where θ^{hist} is the historical fraction of the expenditure that is spent on public investment. In this case, when the cyclical resource revenue increases by one dollar, all windfall is spent, but only the fraction θ^{hist} in extra spending is channeled to investment (the remaining $1 - \theta^{hist}$ falling on government consumption).
- When $\theta = 1$, all cyclical resource revenue is spent, but it falls only on public investment, as prescribed under a BBR-HR (Balanced Budget-Hartwick Rule).

B. The LTGM-NR External Balance

The LTGM-NR External Balance submodel keeps the structure of the Default submodel but accounts for the public sector in more detail and allows for a relationship between the public and private investment via the availability of savings.

The external sector. A key equation in this submodel is the constraint which imposes that aggregate investment must be equal to domestic savings less the current account balance (CAB),

$$i_t = \bar{s}_t^p + s_t^g - cab_t \tag{16}$$

where $\bar{s}_t^p$ and s_t^g denote private and public savings, respectively, and cab_t is the current account balance, all expressed as a share of GDI. Public savings are endogenously determined by the fiscal rule in place (details below), and private savings are assumed to be an exogenous share of GDI. The CAB is financed by (exogenous) foreign direct investment (FDI) or newly created external debt (we abstract from other forms of portfolio investment, as they are less common in developing countries):

$$cab_t = -[d_t - d_{t-1}/(1 + g_t^Y)] - \overline{fdi}_t \tag{17}$$

$$d_t = d_t^p + d_t^g \tag{18}$$

where $\overline{fdi}_t$ denotes FDI in period t, d_t is the outstanding stock of external debt at the end of in period t, both expressed as a share of GDI, and g_t^Y is the net annual growth rate of GDI in period t. Equation (18) decomposes external debt into private and public, d_t^p and d_t^g, respectively.

The relationship between the CAB and fiscal policy is an active debate in the literature and is likely to change substantially from country to country.[18] Much of this literature is about the response of private savings to various shocks (e.g., Loayza et al. 2000), though in our model we assume private savings are simply a fixed share of GDI. In countries with open capital accounts, an increase in public deficits can be funded by foreign savings—a larger current account deficit, resulting in what is known as the "twin deficits."[19] In contrast, if the current account is relatively fixed as a share of GDI—for example, due to thin capital markets or capital controls—then an increase in public savings could free-up financial resources for private investment. Ultimately, we let users choose the degree of crowding in of private investment via a parameter λ:

$$\underbrace{\left(d_t^p - \frac{d_{t-1}^p}{1+g_Y}\right)}_{Private-sector\ deficit} = -\lambda \underbrace{\left(d_t^g - \frac{d_{t-1}^g}{1+g_Y}\right)}_{Fiscal\ deficit} + \bar{d}_t^p \tag{19}$$

where $\bar{d}_t^p$ is the exogenous component of private external debt. Equation (19) implies that a one-dollar fall in public net borrowing crowds in λ dollars of private investment (funded by private net borrowing).[20]

The public sector. The public sector in the External Balance submodel is also more realistic than that in the Default model. The government collects shares τ_R and τ_0 of the resource and non-resource sectors, respectively (equations (20) and (21)). Total revenue is the sum of resource and non-resource revenue (equation (22)). Revenues are used to finance a stream of public expenditure, which is decomposed into government consumption (c_t^g)—which does not affect growth—

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[18] Some empirical studies find that higher budget deficits lead to higher current account deficits, others show evidence of the opposite, or show no significant impact. For a literature review on this subject, see Bussiere et al. (2005) or Cavallo (2005).

[19] Abbas et al. (2011) find that a one percentage point of GDP improvement in the fiscal balance is associated with 1/3 percentage point improvement in the current account, though it is unclear how much of this adjustment is through private savings.

[20] Note that $i_t^p = \bar{s}_t^p + \overline{fdi}_t + \left(d_t^p - \frac{d_{t-1}^p}{1+g_Y}\right)$, where $\bar{s}_t^p$ and $\overline{fdi}_t$ are exogenous. In this case, a fall in the fiscal deficit must be matched by a one-to-one increase in private investment on the left side.

and public investment (i_t^g) (equation (23)). We assume that the split of expenditure between consumption and investment is exogenous and calibrated to match the historical share of public investment in expenditure (see equation (24)):

$$\text{[Non-resource Revenue]:} \; z_t^0 = \tau_0 y_t^0 \tag{20}$$

$$\text{[Resource Revenue]:} \; z_t^R = \tau_R y_t^R \tag{21}$$

$$\text{[Total Revenue]:} \; z_t = z_t^0 + z_t^R \tag{22}$$

$$\text{[Expenditure]:} \; exp_t = c_t^g + i_t^g \tag{23}$$

$$\text{[Public Investment]:} \; i_t^g = \bar{\eta}_t exp_t \tag{24}$$

where $\bar{\eta}_t$ is the share of expenditure falling on public investment (the remaining $1 - \bar{\eta}_t$ on government consumption).

The primary balance is the difference between revenues and non-interest expenditure and represents the government's net borrowing or net lending, excluding interest payments on the outstanding debt (equation (25)). Public savings is defined as revenues less government consumption and represents the amount of resources generated by the government to finance public investment or to pay off the external public debt (equation (26)). Equation (27) describes the evolution of public external debt (as a share of GDI). Each period, the debt grows at the gross rate $(1 + r_{t-1})/(1 + g_t^Y)$, due to payments on the principal and interest on the outstanding bonds, but decreases one-to-one with the primary balance,

$$\text{[Primary Balance]:} \; b_t = z_t - exp_t \tag{25}$$

$$\text{[Public Savings]:} \; s_t^g = z_t - r_{t-1} d_{t-1}^g - c_t^g \tag{26}$$

$$\text{[Public Debt]:} \; d_t^g = \left(\frac{1+r_{t-1}}{1+g_t^Y}\right) d_{t-1}^g - b_t \tag{27}$$

where r_t is the interest rate and b_t is the primary balance.

Following Schmitt-Grohe and Uribe (2003), the country interest rate is the sum of the world interest rate ($r_W > 0$), assumed to be constant, and a spread proportional to the country's total external debt,

$$r_t = r_W + max\,\{0, \psi(d_{t-1} - d)\} \tag{28}$$

where ψ is the debt-elasticity of the interest-rate spread, and d is the long-run external debt to GDI ratio.[21] If $d_{t-1} \leq d$, the government can issue debt at the world interest rate, r_W. If $d_{t-1} > d$, an increase in debt of 1 percent of GDI leads to ψ percentage points increase in the country spread.

Fiscal rules. As in the Default submodel, a fiscal rule determines both the timing and composition of government expenditure. The LTGM-NR External Balance allows for four types of rules: BBR, BBR-HR, SSR, and SSR-HR (see Table 1), up from three types of rules in the Default submodel (where there is no SSR-HR). The External Balance submodel defines the timing aspect of fiscal rules in terms of how the primary balance evolves.

The BBR fixes the headline primary balance as a share of GDI at a target $\bar{b}_t$ (usually fixed but could vary over time exogenously) (see top line of equation (29)). This policy leads to pro-cyclical spending: a one-dollar increase (fall) in resource revenues leads to (almost) exactly one dollar increase (fall) in expenditure.

The SSR mitigates pro-cyclicality by fixing the *structural* primary balance at target $\bar{\bar{b}}_t$. The structural primary balance adjusts for the commodity cycle so that the primary balance tends to increase (decrease) in periods of high (low) commodity prices. More specifically, the structural primary balance is computed based on structural resource revenues, $\breve{b}_t = \breve{z}_t - exp_t$, where the structural revenue, $\breve{z}_t$, is defined as in the default model (equation (15)). The following equation summarizes the fiscal target under BBRs and SSRs:

$$\text{[Fiscal Rule]} \begin{cases} BBR\text{:} \ \ b_t = \bar{b}_t + e_t \\ SSR\text{:} \ \ \breve{b}_t = \bar{\bar{b}}_t + e_t \end{cases} \tag{29}$$

where $e_t = \phi\left(d^g_{t-1} - d_g\right)$ adjusts the target by a fraction ϕ of the deviation of the public debt-to-GDI ratio from its long-run level, d_g, thus ensuring debt sustainability. The parameter ϕ ensures the stability of the public debt and controls its volatility.[22]

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[21] For simplicity, the parameter d is usually set to zero or d_0 but the user can choose other values.

[22] Condition $\phi > (1 + r_W)/(1 + g^Y)$ ensures that the debt-to-GDI ratio fluctuates within bounds around the long-run level. In the limit $\phi \to \infty$ the fiscal rule collapses to a debt-rule: $d^g_t = d_g$.

The BBR and SSR can either keep the composition of spending constant or try to increase investment as in Table 1. As in the Default submodel, the high-investment rule is called a Hartwick Rule (HR), though its application here is more complicated and closer to how it is applied in practice. The principle behind HRs is to prevent extra revenues earned from exhaustible natural resource from being used to finance government consumption. Accordingly, under an HR, all cyclical resource revenue must be invested either in physical or financial assets.[23] The rule is implemented by adding the following inequality to the model:

$$SBI_t \equiv c_t^g / z_t^0 \leq \overline{SBI}_t \qquad \text{(usually, } \overline{SBI}_t = 1\text{)}$$

this inequality states that the Sustainable Budget Index (SBI, the ratio of government consumption to non-resource revenues) must be equal or lower than an exogenously determined threshold $\overline{SBI}_t$ (typically set to one for all t). An SBI_t>1 means that government consumption is being financed at least partially by resource revenues. An SBI_t<1 means that resource revenues are being invested either in physical or financial assets, while consumption is being financed only from non-resource revenues. Capping the SBI_t to one ensures that assets are being preserved (for a detailed discussion, see Lange and Wright 2004).[24]

The standard SBI rule (with $\overline{SBI}_t = 1$) works well for governments with moderate dependence on resource revenues but might be too restrictive for countries where resource revenues represent a large part of the budget. For example, Angola's fiscal oil revenues account for more than 80 percent of total revenues. In this case, it is inevitable that oil revenues are partially consumed to run basic functions of the government. For that reason, the user of the LTGM-NR can choose any positive value for $\overline{SBI}_t$. A possibility is to set $\overline{SBI}_t$ to c_0^g / z_0^0 implying that the SBI_t cannot increase over time (as in section IV.B).[25] Although this configuration does not match perfectly the asset-preservation principle, it prevents the government from increasing consumption in times of high resource revenues.

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[23] Some countries consider spending on education and health as investment in human capital, so the HR would not constrain this type of expenditure.
[24] Botswana is the most celebrated country to incorporate the SBI rule in its fiscal framework for diamond revenues (AfDB 2016).
[25] Another possible application is to set $\overline{SBI}_t$ to match historical SBI values.

On the surface, it seems that the HR has no effect when coupled with an SSR, as this rule already saves most of the transitory windfalls in the short run. However, the HR prevents the government from increasing consumption over time as the financial returns on the invested assets start to improve the fiscal budget.[26]

III. General Baseline Calibration

In this section, we describe how to calibrate the LTGM-NR to a generic oil exporter for a "business-as-usual" baseline simulation. The discussion below intends to be comprehensive and provide a detailed description of the calibration. The general reader that is more interested in how the LTGM-NR works in practice can jump ahead to the Angolan exercise in section IV.

The baseline simulation runs from 2021 to 2050 (though the simulation horizon can be changed). "Flow" variables are usually calibrated to a 20-year average (2000-2019) to reduce year-to-year volatility—also, 2019 is before COVID-19—and slow-moving "stock" variables are calibrated to the most recent year available. Many of the parameters are common to the standard LTGM (indicated with a red cross on Table 2) and so are not discussed further here (see Loayza and Pennings 2018). However, other similar parameters need to be adjusted, as aggregate data sets like the Penn World Tables (PWT) do not include a resource sector. The user has the ability to override the default calibration if they have better data. If the data are missing, we suggest using an average within the same income group or region. Table 2 provides an overview of the calibration and data sources.

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[26] An equilibrium in the External-Balance submodel is defined as a collection of 12 endogenous trajectories $\left\{Y_t^0, Q_t^i, R_t^i, K_t^j, K_t, RGDP_t, Y_t, i_t, i_t^0, i_t^i, MRPK_t^j, MRPK_t^{DS}\right\}$ plus 14 endogenous variables (that are specific to this submodel) $\left\{i_t^p, s_t^g, cab_t, d_t, d_t^p, d_t^g, z_t^0, z_t^R, z_t, exp_t, c_t^g, i^g, b_t, r_t\right\}$ where $j \in \{0,1,\dots,N\}$ with each endogenous variable specified as a function of the exogenous paths $\{\bar{g}_t^{A_j}, \bar{g}_t^h, \bar{g}_t^N, \bar{g}_t^\omega, \bar{g}_t^\varrho, \bar{D}_t^i, \bar{p}_t^i, \breve{p}_t^i, \breve{Q}_t^i\}$ and $\left\{\bar{s}_t^p, \overline{fdi}_t, \bar{\eta}_t, \bar{b}_t\right\}$ as well as initial conditions $\left\{GDP_0, GDP_0^j, K_0, K_0^j, R_0^i, p_0^i, N_0, \omega_0, \varrho_0, d_0^p, d_0^g\right\}$ that satisfy equations (1)—(12) and equations (16)—(29) for every period *t*. For HRs, the SBI inequality must also hold for a specified path $\overline{SBI}_t$ at all periods.

Parameters. The labor share is set to match the average share of labor compensation in non-resource GDP over 2000-2019. This requires adjusting the PWT labor share, which applies to the whole economy, usually less labor intensive than the non-resource sector. Specifically:

$$\beta_t = \beta_{\mathrm{t}}^{PWT} \text{x } GDP_t/GDP_t^0$$

where β_{t}^{PWT} is the share of labor compensation in GDP in period *t*, taken from Penn World Table 10 (PWT10), and GDP_t^0 is non-resource GDP in period *t*. The baseline calibration sets β equal to the average value of β_t over recent years.

To calibrate the resource rents in resource industry i, γ_i, we use information on natural resource rents shares, provided by the Global Trade Analysis Project (GTAP), and averaged over 2004, 2007, 2011, and 2014.[27]

The tax rate in the resource sector, τ_R, is calibrated using data on government natural resource revenues and resource GDP from the IMF's World Commodity Exporters Dataset (IMF-WCE). Specifically, τ_R is set to match the longest available historical average of resource revenues as a share of resource GDP since 2000.

The fiscal rule in the Default submodel requires the calibration of the marginal propensity to invest, θ (see equation (13)). For BBR-HR and SSR, θ is set to one and zero, respectively. For BBR, θ is the average ratio of public investment to total expenditure over 2000-2019, taken from the IMF-WEO and the Investment and Capital Stock Dataset provided by the IMF Fiscal Affairs Department (IMF-FAD).

The External-Balance model requires five additional parameters: the private investment crowd-in parameter λ, the average tax rate on the non-resource economy τ_0, the debt-elastic interest spread ψ, the world real interest rate r_W, and how the budget balance responds to debt ϕ. λ measures the response of private investment to the fiscal balance (equation (19)). Its default value is set to 0.15 (a one-dollar increase in net government borrowing crowds in 15 cents of private investment) based on the average of estimates for both industrial and less developed countries (see Chinn et al. 2011). τ_0 is set to match the average ratio of non-resource revenue

[27] We can map parameter γ_i into the measure of natural resource rents from GTAP as it quantifies the total income that can be generated from the extraction of natural resources, less the cost of extraction, including the return on capital employed on the extractive activity.

to non-resource GDP over 2000 (or most recent historical average). Non-resource revenue is calculated as the difference between total revenue and resource revenue,

Table 2: Baseline set-up of the LTGM-NR:
Selected parameters, initial conditions & trajectories of exogenous variables
(Symbol + indicates the parameter is taken from the standard LTGM)

	Model	SOURCE	TIME SERIES
A. Parameters			
Depreciation rate (δ) +	Both	PWT 10	2000–2019 average
Labor share, non-res. sector (β_0)	Both	PWT 10	2000–2019 average
Resource rents share (γ_i)	Both	GTAP	2004–2014 average
Resource tax rate (τ_R)	Both	IMF-WCE	2000–2016 average
Non-resource tax rate (τ_0)	EBM*	WEO/WCE	2000–2016 average
World real interest rate (r_W)	EBM	FRED	US long-run real interest rate ≈2%
Debt-elasticity of spread (ψ)	EBM	SGU (2003)	Ranges between 0.001 and 0.1
Private investment crowd-in (λ)	EBM	Chinn et al.	Average of industrial and EMDEs
Marginal propensity to invest (θ)	Default	IMF-FAD	BBR: 2000–2017 average of investment to expenditure ratio; SSR: $\theta = 0$; HR: $\theta = 1$
B. Initial conditions			
GDP per capita +	Both	WB-WDI	2020 (or most recent)
Exports of oil (% of GDP)**	Both	UN-CT	2002–2019 average
Capital to GDP ratio: +	Both	PWT 10	2019 or most recent
Non-R sector	Both	Endogenous	Equalize initial MRPK
R sector (oil)	Both	Endogenous	Equalize initial MRPK
Reserves of oil	Both	BP/USGS	Most recent
External public debt (% GDP)	EBM	WB-WDI	2020 (or most recent)
External private debt (% GDP)	EBM	WB-WDI	2020 (or most recent)
C. Trajectory of exogenous variables, 2021–2050			
Price of oil (2010 US$/barrel)	Both	WB-CPD	Most recent or 2000–2019 average
Discoveries of oil (barrels/year)	Both	BP/USGS	2000–2019 average
TFP growth, non-R sector	Both	PWT 10	2000–2019 average
TFP growth, oil sector	Both	--	Country-specific
Human capital+	Both	PWT 10	2000–2019 average
Population+(total, working-age)	Both	ILO	Forecast for 2021–2050
Private savings (% GDI)	EBM	WB-CPD	2000–2019 average
FDI+ (% GDI)	EBM	WB-WDI	2000–2019 average
Private investment (% GDI)	Default	IMF-FAD	2000–2019 average
Exogenous public investment⸸	Both	IMF-FAD	2000–2019 average
Target primary balance (% GDI)	EBM	IMF-WEO	Country-specific
Exogenous private external debt	EBM	--	Country-specific

* EBM denotes External-Balance Model
** For simplicity, we assume that the resource sector has one industry: oil ($N = 1$). In practice, the user can choose up to three industries from the set of commodities: coal, copper, diamond, gas, gold, iron, lead, nickel, oil, silver, tin, and zinc.
⸸ Default model: exogenous public investment (% of GDP); External-Balance Model: exogenous public investment (% of total expenditure).

provided by IMF-WEO and IMF-WCE, respectively.[28] Likewise, non-resource GDP is computed as the difference between GDP and resource GDP. The baseline world annual real interest rate, r_W, is set to two percent, which is in line with the 10-year inflation-indexed US Treasury bond yields averaged over 2000-2019 from the St Louis Federal Reserve Bank Economic Data (Series: WLTIIT).

The baseline debt-elasticity of the interest spread is set to $\psi = 0.1$, which implies that a ten percent of GDI increase in the external debt leads to a one percentage point increase in the country's interest rate.[29] Finally, we set $\phi = 0.05$, which is sufficient to prevent any explosive paths for public debt as $\phi > r_W$.

Initial conditions. GDP for 2020 is taken from World Bank's World Development Indicators (WB-WDI), in constant 2010 U.S. Dollars.[30] In the absence of a data set containing comprehensive information on GDP at the industry level for several commodity-exporting countries, we proxy GDP in resource industry i by exports of the resource good i. More specifically, GDP in industry i is set to match the average value of exports as a share of GDP.[31] The export data is taken from the UN-Comtrade Database (UN-CT), which provides information on export value for all 11 commodities and all 56 countries pre-loaded in the LTGM-NR, with a time series that usually starts in 2002.[32]

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[28] As a complementary data set for government revenues (total, resource and non-resource), we use ICTD/UN-WIDER Government Revenue Dataset (UN-GRD).

[29] The range of estimates for ψ in literature varies widely across countries and papers. For example, while Schmitt-Grohe and Uribe (2003) set $\psi = 0.001$ to match the volatility of the observed current-account-to-GDP ratio for Canada, Schmitt-Grohe and Uribe (2016) estimate $\psi = 1$ for Argentina. We adopt $\psi = 0.1$ as a compromise between these two seemingly extreme estimates.

[30] LTGM-NR spreadsheet since updated to constant 2015 USD.

[31] Initial GDP in industry i is computed as a share of total 2020 GDP. The default method is to use average value of exports in industry i as a share of GDP since 2000. The following expression describes how the default initial real GDP in industry i is computed:

$$GDP_{2020}^{i} = \underbrace{\left(\frac{1}{N}\sum_{t\geq 2000} \frac{\left(p_{2010}^{i}/p_{t}^{i}\right) \text{ x } Exports_{i,t}^{UNCT}}{GDP_{t}^{WDI}}\right)}_{average\ share\ of\ GDP\ in\ industry\ i\ since\ 2000} \text{ x } GDP_{2020}^{WDI}$$

where p_t^i is the real price (2010 U.S. dollars) of resource good i in period t taken from the World Bank Commodity Markets Outlook. $Exports_{i,t}^{UNCT}$ is exports value (current U.S. dollars) of resource good i in period t, and GDP_t^{WDI} is real GDP in period t. As $Exports_{i,t}^{UNCT}$ is measured in current U.S. dollars, it is deflated by p_t^i/p_{2010}^i.

[32] Alternatively, we provide a measure of GDP in industry i derived directly from production data,

As in the standard LTGM, the initial stock of capital-to-GDP ratio is calculated using the most recent observation from PWT 10 (although earlier versions of PWT are also available).[33] The initial capital stock is split across activities to equalize the initial marginal revenue product of capital across $j \in \{0, \dots N\}$.[34]

Information on the initial stock of reserves in industry i is taken from the BP-Energy Dataset for oil, natural gas and coal, and from the U.S. Geological Survey Database (USGS) for mining industries, such as copper, gold, and iron. As a stock variable, initial reserves are set to match the most recent observation (usually 2017).

The External-Balance Model also requires initial public and private external debt. Reliable data on the decomposition of external debt into private and public debt is often unavailable for developing commodity-exporting countries. Hence, in the baseline calibration, we assume that all initial external debt is public, and initial private external debt is zero. This is obviously an extreme assumption, and country-specific data should be used where available. We calibrate public external debt equal to the most recent observation on total external debt taken from WB-WDI.

Trajectory of exogenous variables. The LTGM-NR requires the trajectories of a number of exogenous variables from 2021 until 2050. The key assumption of baseline simulations is that recent trends will continue in the long-term. In this case, we assume that historical averages (such as 2000-2019) will continue until 2050.

The first assumptions are the paths for actual and structural commodity prices. A structural price should reflect its perceived long-term value, so the user could use a long historical average or a proper estimate of the long-run price. For example, in

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$$GDP^i_{2020} = \underbrace{\left(\frac{1}{N}\sum_{t\geq 2000}\frac{p^i_{2010}Q^{BP/USGS}_{i,t}}{GDP^{WDI}_t}\right)}_{Average\ share\ of\ GDP\ in\ industry\ i} \text{x}\ GDP^{WDI}_{2020}$$

where $Q^{BP/USGS}_{i,t}$ is the estimated production of resource good *i* in period *t*. This information is collected from annual reports by BP-Energy Dataset for energy goods (oil, natural gas, and coal) and from the U.S. Geological Survey for mining good (cooper, gold, iron, etc.).

[33] In PWT 10 the capital-to-output ratio is computed as rnna/rgdpna, but in some earlier version (e.g. PWT 8.1) is calculated as rkna/rgdpna.

[34] More specifically, the initial capital stock in activity j must satisfy the following $N + 1$ equations,

$$KY^j_{2020} = \frac{(1-\gamma_j)GDI^j_{2020}}{(1-\beta)GDP^0_{2020} + \sum_{i=1}^{N}(1-\gamma_i)GDI^i_{2020}} \quad for\ j = 0, \dots, N$$

where $GDI^j_{2020} \equiv \left(\bar{p}^j_t / \bar{p}^j_{2010}\right)GDP^i_{2020}$ denotes real GDI in activity j, year t; and $\gamma_0 = \beta$

section IV, we set the structural price of oil to US$50/barrel, which is the estimated unconditional mean of oil prices from 1960 to 2020. The actual path of commodity prices can be set to any value. Again, in section IV, we analyze the consequences of an increase in oil prices from US$50 to US$80/barrel. The default data source for commodity prices is the World Bank's Commodities Prices Dataset (WB-CPD–*The Pink Sheet*), although other sources are available (e.g., USGS and BP-Energy). Also, the structural production of resources is usually assumed to follow N-year moving average of actual production.

Discoveries of natural resources are calibrated using data on annual production and reserves of the resource good i, taken from the BP-Energy (for energy industries) and the USGS (for mining). The time series of discoveries of good i in period t is computed as the change in reserves from period $t-1$ to t plus production in period t (as in equation (3)). In the baseline, the trajectory of discoveries of good i from 2020 to 2050 can be set to match the historical average over the past 20 years. Naturally, predicting future discoveries of natural resources is no trivial task and using historical averages can be misleading. In this case, country-specific data based on experts' knowledge should be used when available.

The LTGM-NR requires paths for future TFP growth in each sector and industry. TFP data at the industry level is usually unavailable for most developing countries. A simple approach is to assume that TFP growth is homogenous across sectors. In this case, we can set TFP growth in each sector/industry equal to the average aggregate TFP growth over 2000-2019, from PWT 10. When available, the user should use information on sectoral TFP growth.[35]

In the Default submodel, private- and public-sector investment data are taken from IMF-FAD, which decomposes total investment into private and public from 1960 to 2017. The paths for private investment, $\bar{\iota}_t^p$, and the exogenous component

[35] For example, the Chilean 2016 Annual Report of the National Productivity Commission documents a large heterogeneity of TFP growth across the copper and non-copper sectors over 2000-2015, with an average fall of one percent in the copper sector and average growth of 1.4 percent in the non-copper sector.

of public investment, $\bar{\iota}_t^g$, are set equal their historical averages over 2000-2017, reflecting "business as usual" investment rates.

In the External Balance submodel, private savings, $\bar{s}_t^p$, are set to match the observed average of private savings (% of GDP) in 2000-2019, computed using data from the IMF-WEO and IMF-FAD. Similarly, the share of public investment in total expenditure, $\bar{\eta}_t$, is set to match the average ratio of public investment to total expenditure over 2000-2017 (also IMF- WEO & IMF-FAD). For simplicity, the default fiscal rule is the BBR-Default, with a zero target for the primary balance ($\bar{b}_t = 0$) and stable external public debt ($d^g = d^g_{2020}$), though the user can refer to the IMF-FAD for actual targets for the fiscal balance and debt in specific countries. Finally, the exogenous component of the private debt, $\bar{d}_t^p$, is set to zero by default.

IV. Application of the LTGM-NR to Angola

In this section, we illustrate how to apply the LTGM-NR in practice with three exercises for Angola. After a brief discussion of the calibration, we assess the baseline business-as-usual growth path over the next three decades (subsection A), how the long-term growth would be affected by higher oil prices (subsection B) and higher oil discoveries (subsection C). Subsection A compares the LTGM-NR-Default with the standard LTGM (submodel 1). Subsections B and C consider all fiscal rules (BBR, SSR, BBR, SSR-HR) and submodels (Default & External Balance). As before, the LTGM-NR only capture supply-side of the economy, and not the short-run response of the economy to shocks via aggregate demand.

Calibration to Angola (Default submodel). The calibration to Angola follows the logic of the generic calibration presented in section III. To keep the description brief, we discuss here only the most important parameters and variables of the Default submodel (for details, see Table 3). We calibrate the model with data up to 2020 but the simulation runs from 2023 to 2050. The years 2021-2022 are excluded from the simulation because the LTGM-NR is not suited to account for the short-term volatility induced by the COVID-19 pandemic. Instead, we refer to the IMF's official forecasts for 2021-2022 (see IMF 2021).

We consider oil as the only resource industry in Angola, as oil accounts for virtually the totality of exports (see Appendix Table 1). Accordingly, we set the oil sector to account for 40 percent of GDI in 2020 (50.8% of GDP due to different base years), which is consistent with the average size of oil exports in Angola over 2007-2015 (UN-CT data).[36] The labor share in the non-oil sector, β, is set to 0.56 to match the average PWT10 aggregate labor share of 0.34 (=0.56 x share of non-oil GDI) over 2015-2019. Due to lack of data for Angola, we use a cross-country average to calibrate the share of oil rents. More specifically, we set γ^{oil} to 1/3 to match the average share of oil rents across large oil exporters, reported by GTAP (see Appendix Figure 1 for details).

The initial capital-to-GDP ratio is set to two, which is a compromise between PWT 8 (1.7 for 2011) and the World Bank's Macro-Fiscal Model Database (2.2 for 2011).[37] The initial oil reserves are set to 9.5 billion of barrels to match the latest estimates for Angola from BP-Energy.

Although PWT reports TFP data for Angola, identifying trend TFP growth is challenging due to the country's extremely volatile business cycle. Instead, we set non-oil TFP growth to one percent to match the average TFP growth in lower-middle-income countries over 2000-2019. The growth rate of oil TFP is set to zero, which is the average TFP growth in large oil exporters over the past two decades (for details, see Appendix Figure 2).

Annual human capital growth is set to 0.7 percent, which was the average growth rate in Angola over 2000-2019 (PWT 10). Moreover, we incorporate the

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[36] More specifically, we want to calibrate oil income as a share of GDI consistently with the data collected in terms of GDP. Angola's oil exports at 2010 prices (~\$80) averaged 50 percent of GDP over 2007-2015. To transform that information into oil income as a share of GDI, we need to adjust both the numerator (oil exports at 2010 prices) and the denominator (real GDP). First, we scale the numerator by 0.625 (50/80) to express oil income in 2020 oil prices (\$50). Second, we compute real GDI in 2020 by scaling real GDP by 0.8 (i.e., $1 + \left(P^{oil}_{2020}/P^{oil}_{2010} - 1\right)$ x $share\ of\ oil\ in\ 2020\ GDP$ $\approx 1 - 0.4$ x 0.5). That is:

$$\frac{Oil\ Income\ in\ 2020\ prices}{GDI_t} = \frac{Oil\ Income\ in\ 2010\ prices}{GDP_t} \text{ x } adjustment = 50\% \text{ x} \frac{0.625}{0.8} = 40\%$$

[37] We opted to use PWT 8 and Macro-Fiscal Model Database for 2011 because the new methodology adopted by PWT 9 and PWT 10 implies extremely high capital-to-GDP ratios for Angola. For example, PWT 10 reports a capital-to-GDP ratio of 6 in 2019. This ratio would lead to a remarkably low marginal product of capital in Angola, which is inconsistent with the country's current level of development.

UN's forecast of demographic trends for Angola, which suggests that population growth will fall from about 3 percent in 2020 to 2 percent by 2050. The working-age population is predicted to rise from 52 percent of total population in 2020 to 58 percent in 2050. Moreover, based on recent trends, the labor force participation rate is assumed to remain constant at around 80 percent of the working-age population until 2050 (WB-WDI).

Table 3: Baseline calibration to Angola (Default model)

Selected Parameters, Initial Conditions & Trajectories of Exogenous Variables

	VALUE	SOURCE	TIME SERIES
A. Parameters			
Depreciation rate (δ)	4.4%	PWT 10	2000-2019 average
Labor share total (β^{PWT})	0.34	PWT 10	2015-2019 average
Labor share, non-oil (β)	0.56	Calculation	2015-2019 average oil share
Oil rents (γ)	0.33	GTAP	Median oil-exporters 2004-14
Government:			
Tax rate in oil sector (τ_R)	0.7	IMF-WCE	2001-2013 average
LR public investment ($\bar{x}$)	6→2% of GDI	IMF-FAD	2015-2017 average
MPI (θ)	0.2	IMF-FAD	BBR: 2000-2017 average; SSR ($\theta = 0$); HR ($\theta = 1$)
Sectoral investment elasticity (μ)	1	Assumption	
B. Initial conditions			
Real GDP per capita	US$2,890*	WB-WDI	2020
Exports of oil ($I = 1$)	50.8% of GDP	UN-CT	2007-2015 average
Capital to GDP ratio:	2	PWT/MFMod	2011
Non-oil sector	1	Endogenous	Equalize initial MRPKs
Oil sector	1	Endogenous	Equalize initial MRPKs
Reserves of oil	9.5 Bi/barrels	BP-Energy	2017
Participation rate, *% of working-age population*			
Male / Female	80.2 / 76.4	WB-WDI	2017
Base year for oil prices	2010		User choice
Initial year of simulation	2020		User choice
C. Trajectory of exogenous variables, 2023-2050			
Price of oil	US$50/barrels	World Bank	Estimated mean 1960-2019
Private investment	20% of GDI	IMF-FAD	2000-2017 average
TFP growth, non-oil sector	1%	PWT 10	Median of LMCs, 2000-19
TFP growth, oil sector	0%	PWT 10	Median oil exporters, 2000-19
Human capital growth	0.7%	PWT 10	2000-2019 average
Oil Discoveries			
Baseline & Discovery Shock	400m bbl/year	BP-Energy	25^{th} percentile 1990-2017
Price shock	Endogenous**	Assumption	
Demographics:			
Population growth	3.4→2.3%	ILO	Forecast for 2023–2050
Working-age population	51→59% pop.	ILO	Forecast for 2023–2050
Population, male	49.1% of pop.	ILO	Forecast for 2023–2050
Participation rate (M & F)	≈0 growth	WB-WDI	2000-2019 average

*Real US Dollars of 2010. **Discoveries are set to keep reserves constant in per worker terms.

We set the government share in the oil sector $\tau_R = 0.7$, which is the historical average of oil revenues (as share of oil GDP) in Angola over 2001-2013 (IMF-WEC, most recent data available). Private investment is set to 20 percent of GDP to match the average observed in 2000-2017 (IMF-FAD). Due to exceptionally large investments in energy infrastructure, public investment has been historically very high in Angola (above 10 percent of GDP), but is expected to fall significantly over time, especially due to the likely decline in oil revenues. Accordingly, we assume that public investment falls from 6 percent of GDI in 2023 to 2 percent by 2050. The marginal propensity to spend under the BBR is set to $\theta_{BBR} = 0.2$, as public investment averaged 20 percent of total government expenditure from 2000 to 2017 (IMF-FAD) (recall $\theta_{SSR} = 0.2$ and $\theta_{HR} = 1$).

Finally, in the baseline we set the actual and structural price of oil to 50 dollars per barrel, which is the estimated unconditional mean estimated with data since 1960 (see Appendix Figure 5 for details). Also, we assume discoveries of 400 million barrels of oil per year in the baseline, equal to the 25th percentile over 1990-2017 (BP-Energy data, see Figure 6 panel A).

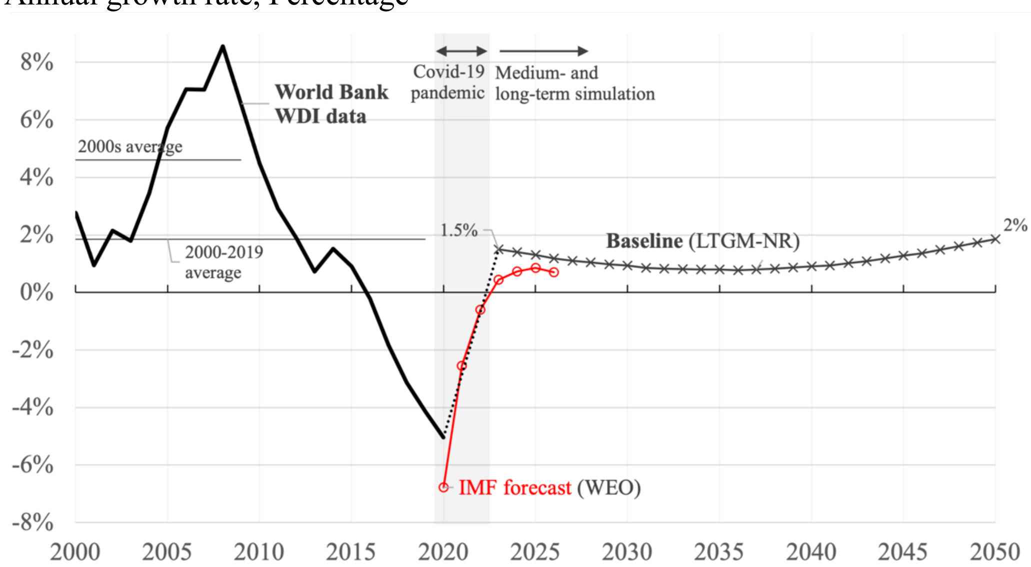
Calibration to Angola (External Balance submodel). For the External Balance submodel, we set the private savings rate and FDI to 19 and 1 percent of GDP, respectively, similar to their averages over the past 20 years. The public investment share in expenditure is set $\eta_t = 0.2$ until 2050 (also the historical average). Finally, we assume that $\bar{b}_t = 0$, which means a balanced budget for the BBR and a structural balance for the SSR. We assume the default private investment crowd-in parameter $\lambda = 0.15$ (Chinn and Ito 2011). For more details of the calibration of the External Balance submodel refer to Appendix Table 4.

A. Baseline Growth in Angola: LTGM-NR versus Standard LTGM

In this subsection, we (i) present the baseline GDP per capita growth simulated by the LTGM-NR-default until 2050, (ii) discuss how this baseline is explained by each of the variables of the model, and (iii) illustrate the importance of using a model with a resource sector (by comparing it with a naïve calibration using the standard LTGM).

Baseline. Figure 1 shows that under the baseline growth path, Angola's potential GDP per capita growth would slow down from 1.5 percent in 2023 to below 1 percent by 2035. After 2035, potential per capita growth would gradually pick up, reaching 2 percent by 2050 (black line with crosses). As a result, GDP per capita would increase from US$2,890 in 2020 to US$4,076 in 2050, a cumulative growth of about 40 percent in 30 years.[38] Note that the baseline growth path is slightly more pessimistic than Angola's historical growth over the past two decades and slightly more optimistic than the April 2021 IMF WEO projections for 2023-2026 (IMF 2021). In terms of headline GDP, growth stands close to 5 percent in 2023 (driven by fast population growth) but also displays a U-shaped trajectory, slowing down in the 2030s and picking up in the 2040s (Appendix Figure 3, Panel A). For more details, see Appendix Table 2.

Figure 1. LTGM Simulation: baseline GDP per capita growth in Angola,
Annual growth rate, Percentage



Note: The solid black line is a five-year moving average of GDP per capita growth in Angola (World Bank WDI).

Growth decomposition. To shed light on the U-shaped dynamics of baseline growth, Figure 2 decomposes the contribution of each macro variable for GDP per

[38] All monetary values are expressed in constant 2010 U.S. dollars.

capita growth in 2023 (initial year of the simulation), 2035 (minimum point for growth), and 2050 (last year).[39] The decomposition shows that the drivers of growth change over time. In 2023, growth is mainly driven by capital accumulation, with total investment generating 3.4 percentage points (ppts) of growth. High population growth and depleting reserves of oil push growth down by 2.3 and 0.5 ppts, respectively. TFP and human capital have only a moderate contribution to growth of 0.7 ppts.

The decomposition for 2035 suggests that the decline in the oil sector is the main reason for the 2023-2035 slowdown—the combined contribution to growth from oil investment and oil reserves falls from +1.5ppts in 2023 to -0.1ppts in 2035. Appendix Figure 3 shows that while GDP per capita grows steadily at 3 percent in the non-oil sector, it falls sharply in the oil sector (Panel B). This decline is mainly driven by depleting oil reserves, projected to halve by 2035 (Panel C). Depleting oil reserves reduces oil output directly but also disincentivizes investment in the sector, reinforcing the initial contraction (Panel D).

Finally, the decomposition shows that the recovery after 2035 is explained by an improvement in demographic trends and a switch of the economy towards the non-oil sector, which has better fundamentals. More specifically, population growth declines substantially, mitigating the negative impact on per capita growth to -1.2ppts (up from -1.8ppts in 2035). In addition, the depletion of reserves (and lack of oil productivity growth) leads gross investment in the oil sector to fall to nearly zero in the long term. As a result, investment in the non-oil sector increases substantially over time, accounting for 2.2ppts of growth in 2050 (versus 1.4ppts in

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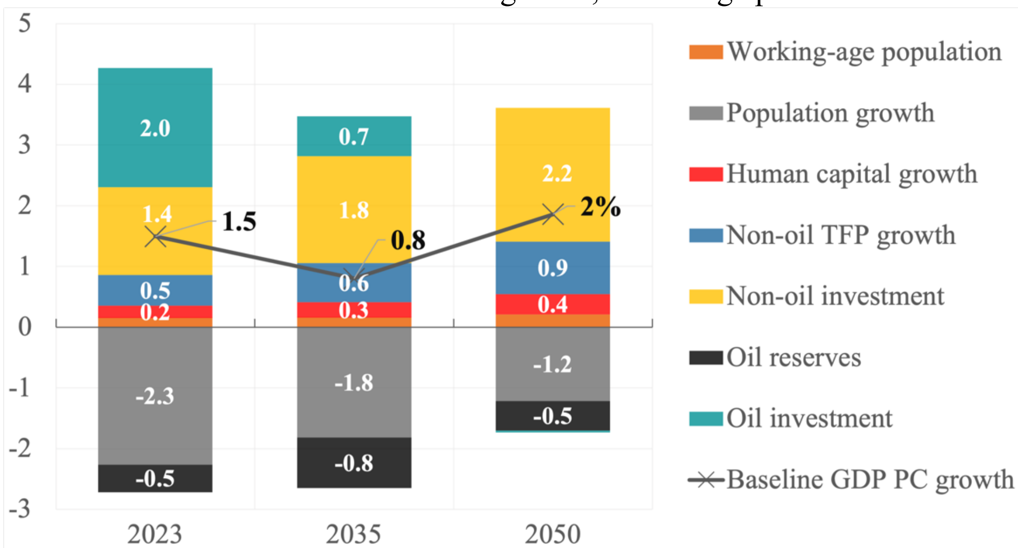
[39] The growth decomposition captures the proximate determinants of growth only—for example, the induced effect of TFP on investment is attributed to the latter, not the former. It is carried out period-by-period by a linear approximation of the effect of each variable on GDP per capita, as in the following expression: $Contribution\ of\ X\ to\ GDP\ per\ capita\ growth\ in\ t$

$$\equiv \frac{\partial_X GDPPC_t \cdot \Delta X_t}{GDP_t} = \left(\frac{GDP_t^0}{GDP_t}\right) \cdot \partial_X GDP_t^0 \cdot \Delta X_t + \left(1 - \frac{GDP_t^0}{GDP_t}\right) \cdot \partial_X GDP_t^{oil} \cdot \Delta X_t$$

For example, GDP per capita in Angola is given by $GDPPC_t = GDPPC_t^0 + GDPPC_t^{oil} = A_t^0 (h_t \omega_t \varrho_t N_t)^{\beta} (K_{t-1}^0)^{1-\beta} / N_t + p_0^{oil} Q_t^{oil} / N_t$. In this case, the contribution of population growth to GDP per capita growth in period t is given by the following simple expression: $\frac{\partial_N GDPPC_t}{GDP_t} \cdot \Delta N_t = \left(-(1-\beta)\frac{GDP_t^0}{GDP_t} - 1\frac{GDP_t^{oil}}{GDP_t}\right)\frac{\Delta N_t}{N_t} = -\left(1 - \beta\frac{GDP_t^0}{GDP_t}\right)\%\Delta N_t$

2023). Moreover, as the non-oil sector accounts for an increasingly large share of the economy, non-oil TFP and human capital increase their contribution to growth: in 2050, they jointly generate 1.3ppts of growth (up from 0.7ppts in 2023).

Figure 2. Year-by-year decomposition of baseline GDP per capita growth,
Contribution of each macro variable to growth, Percentage points



LTGM-NR versus standard LTGM. To illustrate the importance of accounting for sectoral heterogeneity in large resource-rich countries, we compare the baseline growth path implied by the LTGM-NR with the standard "one-sector" LTGM. We produce a naïve calibration of the standard LTGM to be consistent with the LTGM-NR at the aggregate level. Except for the labor share and TFP growth, which are discussed below, the calibration of the two models is essentially the same (see Appendix Table 5 for a full description). It should be noted, however, that an adjusted calibration of the standard LTGM tracks very well growth in the non-resource sector from the LTGM-NR.[40]

Figure 3 shows a large difference between the two models, with the naïve calibration of the standard LTGM (green line) being substantially more optimistic than the NR extension (black line with crosses). Baseline GDP per capita growth

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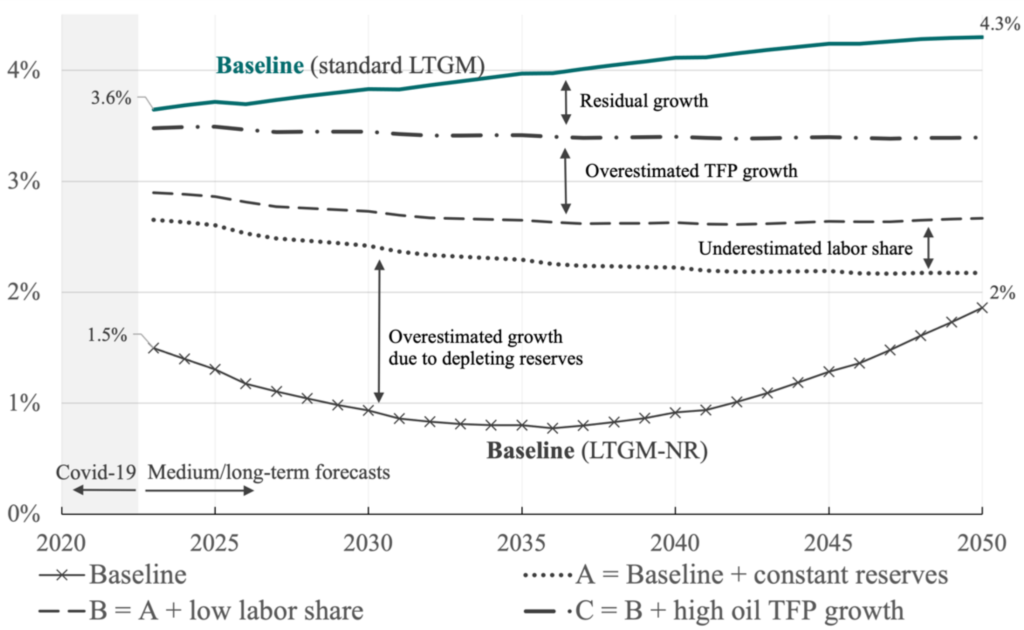
[40] The only adjustment required is to scale up the labor share in the standard LTGM to $\beta = \beta^{PWT} \text{x } GDP_t / GDP_t^0$. In this case, the standard LTGM captures almost perfectly non-oil growth in the LTGM-NR (see Appendix Figure 6).

under the standard LTGM starts at 3.7 percent in 2023 and accelerates to 4.3 by 2050. That speedy growth trajectory is in sharp contrast with the IMF medium-term projections or the recent growth history in Angola.[41]

A combination of three factors explains the "excess" growth implied by the standard LTGM.[42] First, and quantitatively the most important, the standard LTGM does not account for depleting oil reserves. To assess the magnitude of this effect, we run counterfactual "A" with the baseline LTGM-NR but keeping reserves of oil constant over time.[43] The vertical distance between the baseline and counterfactual A shows the excess growth generated by not accounting for depleting reserves. The extra growth averages 1.1ppts over 2023-2050 but shrinks over time as the oil sector becomes less important as a share of GDP (see Appendix Table 3 for details).

Figure 3 Baseline growth simulation: LTGM-NR versus Standard LTGM (naïve calibration) GDP per capita, Percent annual growth rate



Notes: Counterfactual A is equal baseline but with reserves of oil constant over time in per worker terms. Counterfactual B is equal A but with a labor share of 0.34. Counterfactual C is equal B but with one percent oil TFP growth.

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[41] Although Angola reported comparably high growth rates in the 2000s, this growth was related to developments in the oil sector, which is a channel not built-in the standard LTGM. For more details of baseline growth in the standard LTGM, see a growth decomposition in Appendix Figure 7.

[42] The remaining gap between the models is related to factors that are difficult to shut down for analytical purposes, such as the structural differences between the production functions.

[43] In fact, we keep reserves of oil constant *in per worker terms,* which implies that labor productivity does not fall over time due to depleting oil reserves.

Second, the Standard LTGM naïve calibration distorts the impact of growth fundamentals by ignoring the heterogeneity in the labor share across sectors. In the standard LTGM, we set $\beta = 0.34$ to match the labor compensation share in total income from PWT. This particularly low aggregate labor share is the outcome of an economy highly dependent on oil, which is typically a capital-intensive industry.[44] The LTGM-NR specification is more suitable for large oil producers as it allows the user to choose the labor share specific to the non-oil sector. Recall that in the baseline we set $\beta = 0.56$, which is a more conventional value for this parameter—see Appendix Figure 4—but also consistent with the aggregate labor share from PWT (0.56 x initial non-oil share in GDI = 0.34).

The "distorted" low labor share in the naïve calibration of the standard LTGM boosts the effect of investment in physical capital on growth. As physical capital is reproducible, this makes growth much easier. In the limit where the labor share is zero, the standard LTGM becomes an "AK" model that delivers perpetual endogenous growth. To assess the net effect of the low labor share on growth, we run counterfactual B: equal counterfactual A (baseline + constant reserves) but with the labor share lowered to 0.34 (as in the standard LTGM). Not surprisingly, as investment is the main driver in Angola, the net effect is positive and substantial: the distorted labor share leads to extra 0.5ppts of growth on average (as shown by the vertical distance between the dotted and dashed lines).

Third, in the naïve calibration of the standard LTGM we set *aggregate* TFP growth to 1 percent to match the average in lower-middle-income countries over the past two decades. However, there is large heterogeneity in TFP growth across sectors. Recall that in the LTGM-NR we set TFP growth in the *non-oil sector* to one percent but, based on evidence from large oil exporters, we assume no TFP gains in the oil sector (see Appendix Figure 2). To assess the effect of overestimated TFP growth rates, we run counterfactual D: equal counterfactual C (baseline with

[44] Appendix Figure 4 shows that Angola reported the lowest average labor share over 2000-2019 among LMCs (PWT10). It also shows that large oil exporters tend to have low labor shares, which is in line with the empirical evidence of low labor shares in resource industries (see Lebdioui 2021).

constant reserves and low labor share) but with 1 percent oil TFP growth, so aggregate TFP is also 1 percent, as in the standard LTGM. The excessive TFP growth leads to extra 0.6ppts of growth on average over 2023-2050.

B. The Effects of Oil Price Shocks in Angola

In this section, we assess the effects of oil prices on Angola's long-term growth. Note that oil prices shocks have potentially very large effects on short-medium run growth through Keynesian effects. However, those mechanisms are not part of neoclassical growth models like the LTGM-NR, and while important, are not considered here. Moreover, it can be the case that the best fiscal rule for long-run growth produces excessive volatility in the medium term.

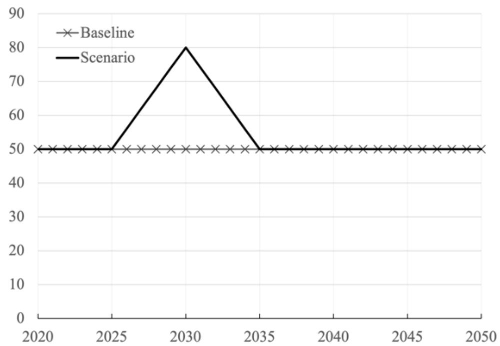
More specifically, we consider a ten-year boom-and-bust cycle, in which the price of oil increases from $50 to $80/barrel from 2025 to 2030 and then returns to $50 by 2035, remaining at that level thereafter (see Figure 4, Panel A). This scenario is assessed relative to the baseline oil price of $50 until 2050. We also set $50/barrel as the reference price to determine structural revenues.[45] To isolate the price effect, we assume that oil reserves are held constant in per worker terms by setting discoveries equal production (both in per worker terms) and the structural production of oil is set equal to actual production.

In this section, most variables are expressed as deviation from the baseline. For example, Panel B of Figure 4 displays incremental GDI as percent of baseline GDI, defined as: $100 \text{ x } [(\text{Scenario real GDI in US\$})/(\text{Baseline real GDI in US\$}) - 1]$ Each line represents the simulation of GDI under a specific fiscal rule (BBR, SSR, BBR-HR, and SSR-HR) and submodel (Default and External Balance). Likewise, panels C, E, and F display incremental GDP, public and private investment, respectively. Finally, panel D shows incremental GDP growth*,* defined as the difference in growth rates between scenario and baseline, in percentage points. Below we briefly discuss the impact of oil prices in each variable. Appendix Table 6 through Appendix Table 8 provide a detailed summary of the results.

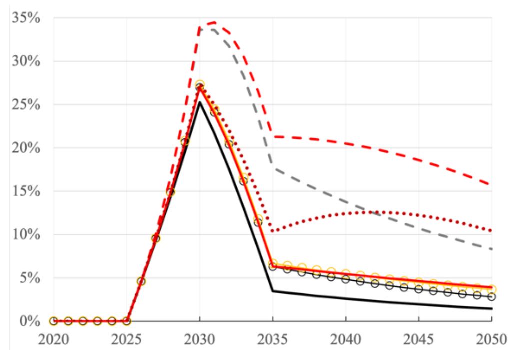
[45] These assumptions are based on the estimated forecast distribution of oil prices, using the World Bank's Commodity Markets Outlook data from 1960 to 2019. The price of $80 is the 75th percentile of the distribution and $50 is the unconditional mean. See Appendix Figure 5 for details.

Figure 4 The effects of oil price shocks in Angola:
Comparison between different fiscal rules and LTGM-NR models
(Incremental = Scenario – Baseline)

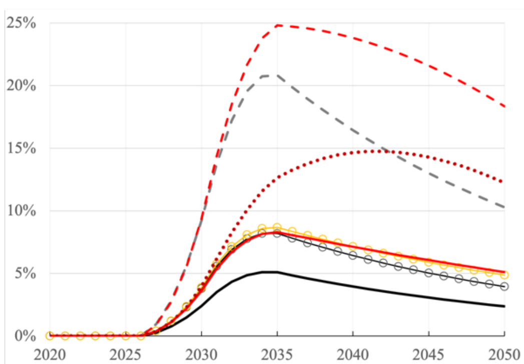
Panel A. Price of Oil,
Real 2010 U.S. Dollars/barrel



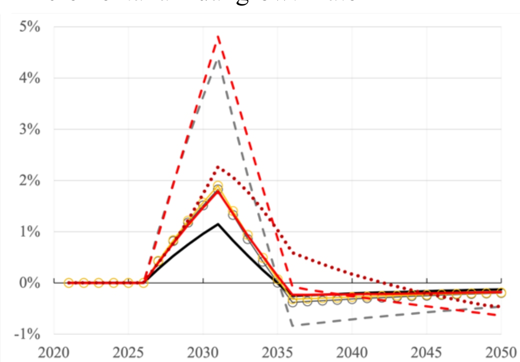
Panel B. Incremental GDI,
Percent of baseline GDI



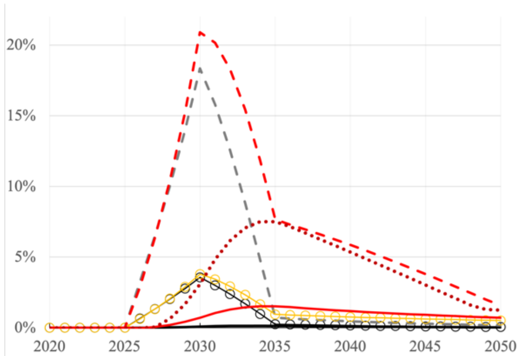
Panel C. Incremental GDP,
Percent of baseline GDP



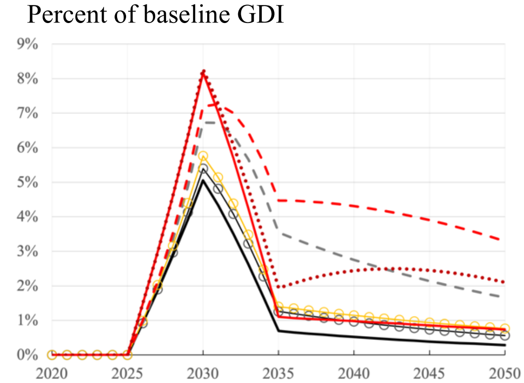
Panel D. GDP (headline or per capita),
Incremental annual growth rate



Panel E. Incremental public investment,
Percent of baseline GDI



Panel F. Incremental private investment,
Percent of baseline GDI



Impact on GDI. The first thing to notice is that, under any specification, the oil price shock leads to large increases in GDI of around 25%-35% (see Figure 4, Panel B). Around 25ppts of this increase is mechanical: oil accounts for around 40 percent of GDI at the start of the simulation, and at the peak, the oil price increases 60 percent (=$80/$50-1), with 40% x 60% ≈ 25%. The excess (depending on the submodel and fiscal rule) is due to (i) reallocation of capital towards the oil sector (which is more productive in dollar terms), and (ii) higher investment. Of course, when the oil price falls over 2030-2035, the effect goes into reverse, and GDI falls sharply.

The paths for different fiscal rules fall into two groups. First, the HR in the Default and BBR-HR in External-Balance models have the largest increase in GDI, hitting around 35 percent above baseline in the early-2030s. This is because the vast majority of commodity revenues are spent on public investment. The second group contains all the other rules: BBR, SSR, and SSR-HR. As discussed below, these rules lead to much smaller increases in investment.

Impact on GDP. In the very short run, the effect of the oil price shock on the level of real GDP is zero because commodity prices do not directly affect GDP (Figure 4, Panel C): for example, by 2026, real GDP is the same as in the baseline, even though oil prices are $6/barrel higher. Instead, GDP only increases after a few years when the productive capacity of the economy expands through higher investment. The flipside of the muted increase in real GDP is the muted *decrease* in GDP when oil prices fall: whereas real GDI (relative to baseline) falls sharply, most measures of GDP only fall slightly and with a delay.[46] The rules that generate a larger increase in GDP are the ones with the higher GDI in Panel B: BBR-HR in both models, with peak GDP about 20-25ppts above the baseline in 2035 (which is five years after the peak oil price). In contrast, the BBR and SSR have a smaller boost to GDP, with a peak of 5-10ppts above the baseline in 2035. The SSR-HR in the Extended Balance submodel generates a boost to GDP that is more sustainable

[46] Note that in reality, large movements in aggregate demand associated with the commodity price shock will cause disruption to domestic non-traded goods production. But these demand side effects are not in this type of neoclassical model.

and lasts well beyond the duration of the price cycle. Moreover, the SSR-HR outperforms the BBR-HR (Default model) in the long run.

While the different rules do have a differential effect on GDP growth rates, it is perhaps less than might be expected (see Figure 4, Panel D). The peak increment to real GDP growth (relative to the baseline) is about 5ppts in 2030 in the BBR-HR (for both submodels). In contrast, the other rules lead to a peak increase in GDP growth of only about 1-2 ppts in 2030—despite a large increase in government revenue (discussed below). Perhaps surprisingly, the increment to real GDP growth continues to be positive as oil prices fall over 2030-2035, and only turns negative after 2035. For the SSR-HR incremental GDP growth remain positive until 2042.[47] Another surprising result is that the BBR and SSR yield almost exactly the same growth path in External-Balance submodel. As we will discuss below, under this calibration with $\eta = 0.2$ and $\lambda = 0.15$ it does not make a large difference if oil revenues are saved or spent under these two rules: each extra dollar of oil revenue becomes 20c of public investment if spent; or 15c of private investment if saved.

Impact on public investment. The oil price shock triggers GDP growth mostly because it leads to higher public investment. In the Default submodel, the increase in public investment as a share of baseline GDI is approximately $\theta \Delta z_t$ (the marginal propensity to invest times the change in revenues). As oil revenues account for almost 30 percent of GDI in 2020 ($\tau_R y_{2020}^{oil} = 0.7\text{x}0.4$), a 60 percent increase in oil prices would boost revenues by nearly 20 percent of baseline GDI (see Appendix Figure 8, Panel A). Under the SSR $\theta = 0$, so public investment is unchanged, which is shown by the black line in Figure 4, panel E. In the BBR-HR, $\theta = 1$, so all the additional revenues are spent, leading to an increase in public investment by around 20 percent of baseline GDI at the peak, as shown by the grey dashed line. Finally, under a BBR $\theta = 0.2$, so 20 cents in the dollar windfall oil revenue is invested, yielding an increase in public investment of about 4 percent of baseline GDI (black circled line). In all cases, public investment nearly returns to baseline levels after the end of the price cycle in 2035. This feature stems from the assumption that

[47] The reason is that oil prices have no direct effect on real GDP—only an indirect effect via investment, which continues to be elevated as long as oil prices are above their steady-state level.

structural production of oil equals actual production, implying that $\theta(z_t^R - \check{z}_t^R) = \theta(P_t^{oil} - \$50)Q_t^{oil} = 0$ for all θ after 2035 as P_t^{oil} returns to \$50 (equation (13)).

In the External-Balance submodel, the paths for public investment under the BBR and SSR are fairly similar to their Default submodel counterparts. However, the deviations of BBR-HR and SSR-HR are worth noting, as they both lead to persistently higher public investment, though for different reasons.

Under the BBR-HR, the oil sector expands rapidly during the oil price cycle, generating higher oil revenues that are reinvested under the BBR-HR. Though this effect is present in other simulations, it is especially strong for the BBR-HR. For example, in 2035, incremental revenues stand at 10 ppts of baseline GDP under the BBR-HR, more than double the value generated by the BBR or SSR (see Appendix Table 6 and Appendix Figure 8).

The SSR-HR (red dotted line) generates high public investment for many years after the end of the oil cycle (extra 5ppts on average in 2036-2050, see Appendix Table 8). This is because the extra interest savings from paying down government debt are recycled into the budget, releasing extra resources for higher public investment (for details, see Appendix Figure 8).

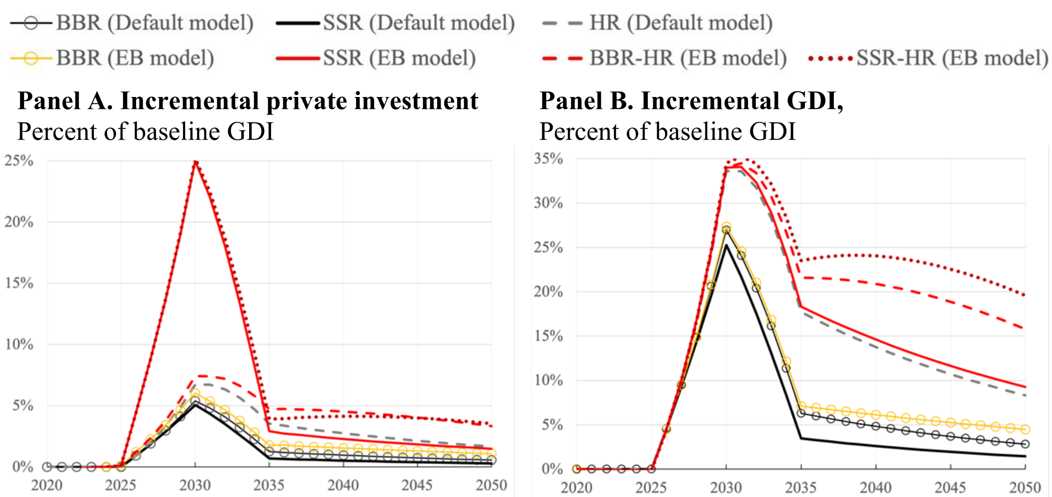
Impact on private investment and spillovers. Private investment increases in all simulations, though can be much larger in the External Balance submodel than the Default submodel. In the Default submodel, private investment is 20% of GDI, so as GDI increases by 25-40 percent, private investment increases by 5-8ppts of baseline GDI (Figure 4, Panel F). In the External Balance model we assume that if the public sector runs a surplus of one dollar, this will free up λ dollars for private investment. However, it is only the SSR and SSR-HR that generate substantial changes in the budget balance, and hence it is only these two rules that feature larger increases in private investment with $\lambda > 0$.[48] With our default calibration of $\lambda = 0.15$, private investment increases further to peak at 8ppts above baseline by 2030 in the SSR and SSR-HR.

[48] Under the BBR the budget is balanced, so there is almost no increase in the fiscal surplus. In practice there is sometimes a small increase in fiscal savings, because the BBR is specified as a share of GDI, which increases with an oil price shock.

The combined effect of higher public and private investment provides a strong engine of growth in Angola because of the low initial capital-to-output ratio (2), which makes GDP growth sensitive to investment rates.

Extension: a higher investment crowd-in λ parameter. Now we discuss an alternative parametrization with a higher level of crowding-in of private investments, $\lambda = 1$ (up from the default of $\lambda = 0.15$). When $\lambda = 1$, the current account balance is effectively fixed as a share of GDI. As private savings and FDI are also assumed to be fixed shares of GDI, then an extra dollar of fiscal surplus ends up funding an extra dollar of private investment (up from 15 cents in the baseline). [49]

Figure 5 The effects of oil price shocks in Angola:
High private investment crowd-in ($\lambda = 1$) (fixed CAB as share of GDI)



This calibration has little effect on BBR and BBR-HR rules, because they generate no additional fiscal surplus, but allows the SSR and SSR-HR rules to generate much higher investment and growth rates. One can see this on panel A of Figure 5, where private investment increases by 25ppts of baseline GDI for the SSRs with $\lambda = 1$ in 2030, rather than 8ppts in the simulations with the default

[49] In practice, the extra private investment could be supplied by a reduction of credit rationing and easing of loan conditions or lower interest rates when domestic banks no longer purchase so many government bonds (though we do not model this). The calibration also applies most effectively to countries with very closed current accounts. In contrast, a small λ is more appropriate for countries open to capital flows.

calibration of $\lambda = 0.15$. This means the path for GDI with either SSR in the External-Balance submodel is now similar to the BBR-HR (Figure 5, panel B), achieving a 35ppt increase in GDI by 2030 (though through higher private rather than public investment). Moreover, the SSR-HR achieves the highest growth path—also peaks at 35ppts of baseline GDI in 2030—but outperforms the other rules in the longer-term. Finally, note that different from the default calibration, the BBR and SSR yield very different trajectories with $\lambda = 1$.

C. The Effects of a Large Discovery of Oil in Angola

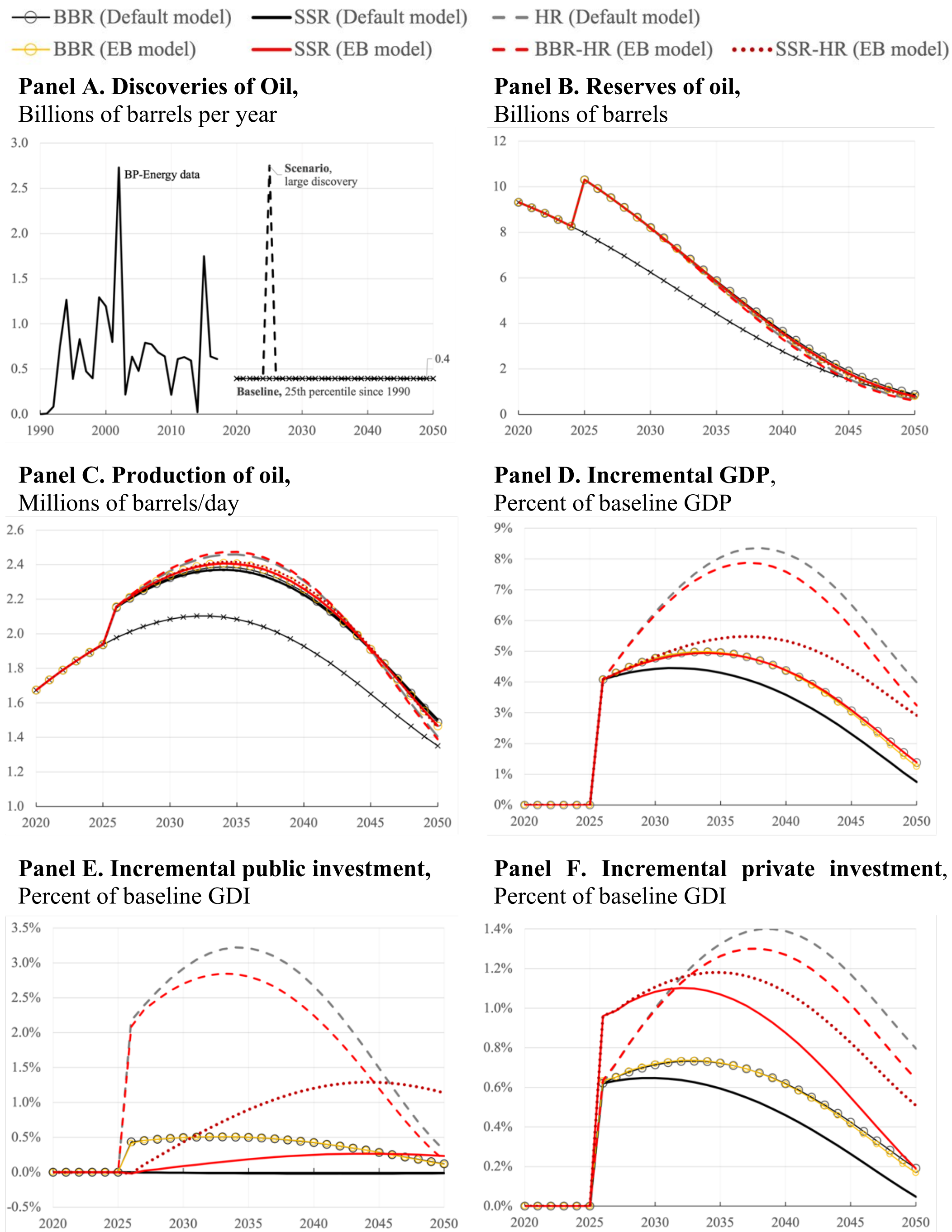
In this section, we assess the effects of a large discovery of oil in Angola. Specifically, we assume a discovery of 2.7 billion barrels of oil in 2025, calibrated to match the 2002 discovery, which was the largest over the last 30 years. In all other years, as in the baseline, we assume discoveries of 400 million barrels, equal to the 25^{th} percentile of the distribution over 1990-2017 (see Figure 6 panel A).

Note that in reality, discoveries often become useable "reserves" over a period of many years. While we assume an immediate transition here for analytical purposes—to clarify the effect of the shock—we advise that users of the LTGM-NR smooth the discovery over several years.

To isolate the effect of the 2025 discovery, we assume constant oil prices at $50/barrel, as in the baseline. This last assumption means that real GDI and real GDP—which had large differences in section B—will have similar growth profiles, so in this subsection we only report results for GDP. Finally, to allow the discovery shock to generate cyclical oil revenues—and, hence, different spending profiles across fiscal rules—we set structural (long run) oil production equal to the baseline oil production.[50]

[50] Note that this assumption is different from section IV.B, where structural production of oil was set equal to actual production. That assumption was designed to isolate the effect of oil price shocks on the government spending profile. However, we now want to explore the effects of higher oil output across different fiscal rules, keeping oil prices constants. This can be done assuming that structural production is equal the baseline, although similar results would be achieved by assuming that structural production follows an N-year moving average of actual production.

Figure 6 The effects of oil discoveries in Angola:
Comparison between different fiscal rules and LTGM-NR models*
(Incremental = Scenario – Baseline)



The first-order effect of an oil discovery is to increase reserves and, hence, production: a 1 percent increase in reserves leads to $\gamma = 1/3$ percent increase in oil production (see equation (2)). Panel B of Figure 6 shows that the 2025 discovery raises oil reserves from 8 billion to 10 billion barrels, a 25 percent increase relative to baseline. In 2026, higher reserves boost oil production to nearly 2.2 million barrels per day, a 9 percent increase relative to baseline ($\approx \gamma$ x $\%\Delta R^{oil}_{2025}$, see Panel C) in almost all models. As oil accounts for about half of Angola's economy, the increase in oil production boosts total GDP by about four percentage points in 2026 (see Panel D).

After 2026, the impact on growth depends mostly on public investment, which, in turn, depends on how the government spends/saves the windfall revenue generated by the discovery (Panel E). The shock leads to extra revenues of 2-3 percent of baseline GDI until 2040 but declines sharply after that due to depleting oil reserves (see Appendix Figure 9).[51] This path is very similar across models and rules, except from the BBR-HR, which generate higher growth and, hence, higher oil revenues in the 2030s.

The BBR-HRs (both models) are the only rules that yield a faster growth rate for several years after the discovery shock. As above, the BBR-HRs invest all windfall oil revenue, which increases GDP growth (oil and non-oil) and private investment vis-à-vis the baseline.[52] In contrast, the simple BBRs (either model), invest only 20 percent of the windfall, leading to a modest impact on growth after 2026.

The SSRs also generate a relatively small boost to public investment and long-term growth because the extra revenues are mostly classified as cyclical and so are saved. The SSR in the Default submodel generates the lowest growth path after 2026 as public investment remains constant and private investment increases only modestly vis-à-vis baseline. As above, the SSRs in the External-Balance submodel also generate a small amount of crowding-in of private investment (15 cents per dollar of fiscal surplus, see Panel F).

Over the long term, the differences in public and private investment accumulate to modest differences in the capital stock and GDP across the different fiscal rules. In the BBR-HRs (both submodel), GDP is about 8 percent above baseline by 2040,

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[51] For example, in 2026, oil revenues increased by just above 2 percent of baseline GDI: a 9% increase in oil production x 35% oil as share of total GDI in 2025 x 70% oil tax rate.

[52] Recall that private investment increases in absolute terms because it is a fixed fraction of GDI, which increases relative to baseline after 2025 due to the discovery.

which is double the mechanical increase in 2026 (see Panel D). However, by 2040, GDP under the non-HRs are similar to the initial shock in 2026 of 4 ppts above baseline (variation 3.5-4.5ppts, depending on the rule). Finally, as above, the SSR-HR recycles interest savings into higher public investment. While the increase in investment (public and private) does boost the productive capacity of the economy under these rules, a higher rate of extraction reduces reserves, which is roughly offsetting. Consequently, by 2040 around 3/4 of the initial discovery has been depleted (see Panel B). See Appendix Table 9 and Appendix Table 10 for further details.

V. Conclusion

This paper develops the Natural Resource extension of the World Bank's Long-Term Growth Model (LTGM-NR), which evaluates the effects of commodity price shocks and discoveries of natural resources on medium- and long-term growth in resource-rich economies. The LTGM-NR augments a relatively standard neoclassical model (the LTGM) by adding a resource sector and a government whose fiscal rule determines how to spend or save resource revenues. The model is designed to be simple enough so that its mechanisms are clearly understood, and its solution can be implemented in a spreadsheet (without macros). The accompanying toolkit is preloaded with data for 56 commodity-rich economies and 11 resource industries and can be adjusted to the needs of users.

In a calibrated version of the model to Angola, we find that population growth, depleting oil reserves, and a reallocation of the economy towards the non-oil sector leads to a medium-term growth slowdown and subsequent recovery in the long term. This non-linearity is difficult to capture in standard one-sector neoclassical models. Failing to account for sectoral heterogeneity, naïve calibrations could lead to large differences in growth projections.

Next, we find that an increase in commodity prices can substantially affect real income (real GDI), though only while prices remain elevated. In contrast, the effect on real GDP is smaller and delayed because only the increase in the volume of production is counted (not the price effect). The boost to GDP is more persistent—outlasting the price boom—but its size depends on the government fiscal rule. Not surprisingly, Hartwick rules that invest all extra resource revenues generate the largest increase in GDP. Structural surplus rules (SSRs) can potentially yield large growth rates in countries where higher public surpluses crowd-in private investment. However, in our default calibration for Angola, the crowd-in is small and so SSRs perform similarly to balanced-budget rules where the fraction of extra spending on public investment is equal to the historical average.

In contrast with price shocks, when resource discoveries become available for production there is the same immediate boost to GDP and GDI. The size of the gain largely depends on the share of resource rents in resource production. The subsequent evolution of GDP depends on the fiscal rules in a similar way to price shocks.

In closing, it is important to mention several caveats. First, the LTGM-NR is a simple neoclassical growth model and so omits some mechanisms connecting the commodity sector and growth that are outside this framework. Most important is the lack of an aggregate demand side, which means there is no stimulatory effect of extra commodity-related spending on the local economy in the short run. Our model also lacks channels through which commodity wealth might reduce long-term growth, such as Dutch Disease or a growth-sapping political economy.

Second, governments may have many different objectives in designing fiscal rules governing the use of resource revenues, beyond long-term growth. Other objectives include intergenerational equity (particularly in countries with limited reserves), consumption smoothing, and reducing business-cycle volatility. Often these objectives are conflicting. For example, our rule that delivers the fastest long-term growth (a Hartwick rule) can also result in procyclical spending that exacerbates the business cycle in the short term. As such, the findings in this paper regarding the ranking of different rules reflect only one dimension of performance – medium and long-term growth – and are not a blanket recommendation.

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Technical Appendix for

"Assessing the Effects of Natural Resources on Growth:

An Extension of the World Bank Long-term Growth Model"

Appendix Table 1: Commodity Exports in Resource-rich Economies, 2008-2012 Average

				All commodities		Largest commodity		2nd largest commodity	
N	ISO	Country	Region	% of Exports	% of GDP	Type	% of GDP	Type	% of GDP
Cross-country average			**ALL**	**66%**	**24%**	**ALL**	**18%**	**ALL**	**3%**
1	AUS	Australia	EAP	70%	12%	iron	3%	coal	3%
2	IDN	Indonesia	EAP	57%	12%	coal	3%	natural gas	2%
3	MNG	Mongolia	EAP	74%	30%	copper	14%	gold	6%
4	MYS	Malaysia	EAP	30%	23%	oil	8%	natural gas	6%
5	PNG	Papua New Guinea	EAP	33%	9%	palm oil	3%	copper	2%
6	VNM	Vietnam	EAP	28%	18%	oil	7%	rice	3%
7	AZE	Azerbaijan	ECA	95%	47%	oil	46%	natural gas	1%
8	KAZ	Kazakhstan	ECA	83%	36%	oil	29%	copper	2%
9	NOR	Norway	ECA	73%	23%	oil	13%	natural gas	8%
10	RUS	Russian Federation	ECA	75%	18%	oil	12%	natural gas	3%
11	BOL	Bolivia	LAC	76%	29%	natural gas	16%	zinc	4%
12	CHL	Chile	LAC	64%	21%	copper	18%	gold	1%
13	COL	Colombia	LAC	71%	11%	oil	6%	coal	2%
14	ECU	Ecuador	LAC	77%	21%	oil	15%	banana	3%
15	JAM	Jamaica	LAC	34%	4%	oil	2%	aluminum	1%
16	MEX	Mexico	LAC	20%	6%	oil	4%	gold	1%
17	PER	Peru	LAC	72%	18%	copper	6%	gold	5%
18	SUR	Suriname	LAC	12%	6%	oil	5%	rice	1%
19	TTO	Trinidad and Tobago	LAC	69%	37%	oil	20%	natural gas	14%
20	VEN	Venezuela, RB	LAC	91%	24%	oil	23%	aluminum	0%
21	ARE	United Arab Emirates	MENA	53%	39%	oil	27%	gold	5%
22	BHR	Bahrain	MENA	87%	49%	oil	38%	aluminum	7%
23	DZA	Algeria	MENA	98%	36%	oil	22%	natural gas	14%
24	EGY	Egypt, Arab Rep.	MENA	48%	6%	oil	3%	natural gas	1%
25	IRN	Iran, Islamic Rep.	MENA	73%	15%	oil	13%	natural gas	1%
26	IRQ	Iraq	MENA	100%	42%	oil	42%	potassium	0%
27	KWT	Kuwait	MENA	94%	54%	oil	52%	natural gas	2%
28	LBY	Libya	MENA	98%	54%	oil	51%	natural gas	3%
29	OMN	Oman	MENA	75%	48%	oil	40%	natural gas	5%
30	QAT	Qatar	MENA	88%	54%	natural gas	33%	oil	20%
31	SAU	Saudi Arabia	MENA	88%	46%	oil	44%	natural gas	1%
32	SYR	Syrian Arab Republic	MENA	46%	10%	oil	9%	aluminum	0%
33	TUN	Tunisia	MENA	23%	9%	oil	6%	dap	1%
34	YEM	Yemen, Rep.	MENA	89%	20%	oil	19%	natural gas	2%
35	**AGO**	**Angola**	**SSA**	**99%**	**60%**	**oil**	**59%**	**diamond**	**1%**
36	BFA	Burkina Faso	SSA	82%	12%	gold	9%	cotton	3%
37	BWA	Botswana	SSA	85%	31%	diamond	26%	nickel	3%
38	CIV	Côte d'Ivoire	SSA	67%	28%	oil	11%	cocoa	10%
39	CMR	Cameroon	SSA	72%	8%	oil	3%	cocoa	2%

40	COG	Congo, Rep.	SSA	76%	59%	oil	57%	natural gas	1%
41	GAB	Gabon	SSA	88%	47%	oil	46%	sawn wood	1%
42	GHA	Ghana	SSA	85%	23%	gold	11%	cocoa	4%
43	GIN	Guinea	SSA	82%	16%	aluminum	9%	gold	6%
44	MLI	Mali	SSA	89%	17%	gold	14%	cotton	2%
45	MOZ	Mozambique	SSA	61%	14%	aluminum	9%	tobacco	1%
46	NAM	Namibia	SSA	38%	20%	diamond	10%	zinc	4%
47	NER	Niger	SSA	13%	2%	gold	1%	oil	1%
48	NGA	Nigeria	SSA	94%	24%	oil	21%	natural gas	1%
49	SDN	Sudan	SSA	90%	12%	oil	10%	gold	2%
50	SEN	Senegal	SSA	38%	6%	oil	3%	gold	1%
51	SLE	Sierra Leone	SSA	48%	2%	tin	1%	cocoa	1%
52	TGO	Togo	SSA	22%	5%	cotton	1%	dap	1%
53	UGA	Uganda	SSA	44%	4%	coffee	2%	oil	0%
54	ZAF	South Africa	SSA	42%	10%	platinum	3%	coal	2%
55	ZMB	Zambia	SSA	83%	28%	copper	25%	maize	1%
56	ZWE	Zimbabwe	SSA	61%	18%	nickel	6%	tobacco	4%

Notes: A country is defined as commodity exporter if, on average in 2008-2012, commodity exports accounted for at least 5% of GDP or 1/3 of total exports. **Source**: Author's calculations based on exports data from UN-Comtrade Database and GDP data from the World Bank's World Development Indicators.

Appendix Table 2: LTGM-NR Simulation - Baseline GDP per capita growth in Angola

	Annual growth rates				**Real 2010 US$**		
	Average 2023-2050	**2023**	**2035**	**2050**	**2020**	**2035**	**2050**
I. GDP per capita							
Total	1.1	1.5	0.8	1.9	2,890	3,417	4,076
Non-oil	3.1	3.1	3.1	3.1	1,421	2,246	3,558
Oil	-3.7	-0.2	-3.3	-6.1	1,469	1,171	518
II. GDI per capita							
Total	1.67	1.9	1.4	2.3	2,350	2,987	3,886
Non-oil	3.1	3.1	3.1	3.1	1,421	2,246	3,558
Oil	-3.7	-0.2	-3.3	-6.1	930	741	328

Appendix Table 3. Baseline Growth Simulations - LTGM-NR versus Standard LTGM

	GDP per capita growth, %			
Models:	Average 2023-50	2023	2035	2050
Baseline (Standard LTGM)	3.9	3.6	4.0	4.3
Baseline (LTGM-NR)	1.1	1.5	0.8	1.9
A. Baseline + constant reserves	2.2	2.7	2.3	2.2
B. Baseline + constant reserves + low labor share	2.7	2.9	2.7	2.7
C. Baseline + constant reserves + low labor share + high TFP growth	3.3	3.5	3.4	3.4

Appendix Table 4: Baseline calibration to Angola: External Balance Model

Selected Parameters, Initial Conditions and Trajectories of Exogenous Variables

	VALUE	UNIT	SOURCE	TIME SERIES
A. Private savings	19%	% of GDI	WB-WDI	2000-2017 average
B. External Balance				
Private savings	19%	% of GDI	WB-WDI	2000-2017 average
Foreign direct investment	1%	% of GDI	WB-WDI	2000-2019 average (w/o outliers)
Private external debt:				
Initial value	0%	% of GDI	No data	Default when data is not available
Long run	0%	% of GDI	No data	Default when data is not available
Exogenous component	0%	% of GDI	No data	Default when data is not available
C. Fiscal Balance				
Target primary balance	0%	% of GDI		Balanced budget
Public investment	20%	% of Expenditure	IMF-FAD	2000-2019 average
Parameters:				
Govt. share in non-oil, τ_0	10%	% of non-oil GDP	IMF-WCE	2000-2013 average
World interest rate, r_W	2%	Annual	US FED	Summary of Economic Projections
Interest spread param., ψ	0.05	p.p.	SGU (2003)	Compromise between 0.01 and 0.1
Adjustment parameter, ϕ_D	0.05			$\phi_D > r_W$ to induce debt stability
Crowd-in parameter, λ	0.15			Chinn et al. (2011)
Public debt:				
Initial	60%	% of GDI	WB-WDI	2019
Long run	0%	% of GDI		Default

Appendix Table 5: Baseline calibration of the Standard LTGM (Submodel 1) to Angola

Selected Parameters, Initial Conditions and Trajectories of Exogenous Variables

(Symbol + indicates the calibration is equal the LTMG-NR)

	VALUE	UNIT	SOURCE	TIME SERIES
A. Parameters				
Depreciation rate (δ)+	4.3%	Annual	PWT 10	2000-2019 average
Labor share (β)	0.34	[0,1]	PWT 10	2015-2019 average
B. Initial conditions				
GDP per capita+	2,890	Real, 2010 US$	WB-WDI	2020
Capital to GDP ratio+	2	$[0, \infty)$	PWT/MFMod	2011
Demographics:				
Participation rate, male+	80.2%	% working-age pop	WB-WDI	2017
Participation rate, female+	76.4%	% working-age pop	WB-WDI	2017
C. Trajectory of exogenous variables, 2023-2050				
Investment*	≈21%	% of GDP	IMF-FAD	2000-2017 average
TFP growth	1%	Annual growth rate	PWT 10	Median of LMCs in 2000-2019
Human capital growth+	0.7%	Annual growth rate	PWT 10	2000-2019 average
Demographics:				
Population+	3.4→2.3%	Annual growth rate	ILO	Forecast for 2023–2050
Working-age population+	51→59%	% total population	ILO	Forecast for 2023–2050
Population, male+	49.1%	% total population	ILO	Forecast for 2023–2050
Participation rate, male+	≈ 0%	Annual growth rate	WB-WDI	2000-2019 average
Participation rate, female+	≈ 0%	Annual growth rate	WB-WDI	2000-2019 average

* Investment is set to 21% of GDP, which is the implied investment rate *as percent of GDP* in the LTGM-NR applications.

Appendix Table 6: LTGM-NR simulation for Angola: Effects of a 10-year boom-and-bust oil price shock on GDP

	Average growth rate			Real 2010 US$		
	ALL 2026-2050	Oil cycle 2026-2035	Post-cycle 2036-2050	2020	2035	2050
I. GDP per capita						
Baseline	2.4	2.7	2.4	2,890	4,173	5,783
Incremental growth (p.p.):						
BBR (Default)	0.2	0.9	-0.3	//	4,515	6,011
SSR (Default)	0.1	0.6	-0.2	//	4,386	5,921
HR (Default)	0.4	2.2	-0.7	//	5,042	6,378
BBR (EB)	0.5	1.2	0.1	//	4,649	6,565
SSR (EB)	0.5	1.2	0.2	//	4,632	6,582
BBR-HR (EB)	1.0	2.8	0.0	//	5,329	7,347
SSR-HR (EB)	0.8	1.6	0.3	//	4,805	6,969
II. Non-oil GDP per capita						
Baseline	3.1	3.3	3.1	1,421	2,223	3,416
Incremental growth (p.p.):						
BBR (Default)	0.0	0.0	0.0	//	2,226	3,400
SSR (Default)	-0.1	-0.2	0.0	//	2,184	3,373
HR (Default)	0.1	0.9	-0.4	//	2,394	3,505
BBR (EB)	0.3	0.2	0.3	//	2,272	3,639
SSR (EB)	0.3	0.2	0.3	//	2,267	3,649
BBR-HR (EB)	0.6	1.3	0.1	//	2,488	3,894
SSR-HR (EB)	0.4	0.5	0.4	//	2,327	3,794
III. Oil GDP per capita						
Baseline	1.5	2.0	1.4	1,469	1,950	2,367
Incremental growth (p.p.):						
BBR (Default)	0.4	1.8	-0.4	//	2,289	2,611
SSR (Default)	0.3	1.4	-0.3	//	2,202	2,547
HR (Default)	0.8	3.5	-0.8	//	2,648	2,873
BBR (EB)	0.9	2.2	0.1	//	2,376	2,926
SSR (EB)	0.9	2.2	0.2	//	2,365	2,933
BBR-HR (EB)	1.6	4.3	0.0	//	2,841	3,453
SSR-HR (EB)	1.2	2.7	0.4	//	2,477	3,175

Appendix Table 7: LTGM-NR simulation for Angola: Effects of a 10-year boom-and-bust oil price shock on GDI

	Average growth rate			Real 2010 US$		
	ALL 2026-2050	Oil cycle 2026-2035	Post-cycle 2036-2050	2020	2035	2050
I. GDI per capita						
Baseline	2.5	2.8	2.5	2,350	3,457	4,914
Incremental growth (p.p.):						
BBR (Default)	0.1	0.7	-0.2	2,350	3,674	5,052
SSR (Default)	0.1	0.4	-0.1	2,350	3,577	4,985
HR (Default)	0.3	1.9	-0.6	2,350	4,069	5,323
BBR (EB)	0.5	1.0	0.2	2,350	3,775	5,490
SSR (EB)	0.5	0.9	0.2	2,350	3,763	5,504
BBR-HR (EB)	0.9	2.5	0.0	2,350	4,286	6,078
SSR-HR (EB)	0.7	1.3	0.3	2,350	3,894	5,802
II. Non-oil GDI per capita*						
Baseline	3.1	3.3	3.1	1,421	2,223	3,416
Incremental growth (p.p.):						
BBR (Default)	0.0	0.0	0.0	//	2,226	3,400
SSR (Default)	-0.1	-0.2	0.0	//	2,184	3,373
HR (Default)	0.1	0.9	-0.4	//	2,394	3,505
BBR (EB)	0.3	0.2	0.3	//	2,272	3,639
SSR (EB)	0.3	0.2	0.3	//	2,267	3,649
BBR-HR (EB)	0.6	1.3	0.1	//	2,488	3,894
SSR-HR (EB)	0.4	0.5	0.4	//	2,327	3,794
III. Oil GDI per capita						
Baseline	1.5	2.0	1.4	930	1,234	1,497
Incremental growth (p.p.):						
BBR (Default)	0.4	1.8	-0.4	//	1,448	1,652
SSR (Default)	0.3	1.4	-0.3	//	1,393	1,611
HR (Default)	0.8	3.5	-0.8	//	1,675	1,817
BBR (EB)	0.9	2.2	0.1	//	1,503	1,851
SSR (EB)	0.9	2.2	0.2	//	1,496	1,856
BBR-HR (EB)	1.6	4.3	0.0	//	1,797	2,184
SSR-HR (EB)	1.2	2.7	0.4	//	1,567	2,008

* Non-oil GDI per capita is equal non-oil GDP per capita

Appendix Table 8: LTGM-NR simulation for Angola: Effects of a 10-year boom-and-bust oil price shock on investment

	Percent of GDI		
	ALL 2026-2050	Oil cycle 2026-2035	Post-cycle 2036-2050
I. Investment, Percent of GDI			
Baseline	24%	25%	23%
BBR (Default)	24%	26%	23%
SSR (Default)	23%	24%	23%
HR (Default)	26%	31%	23%
BBR (EB)	27%	27%	26%
SSR (EB)	27%	27%	26%
BBR-HR (EB)	31%	35%	28%
SSR-HR (EB)	29%	29%	28%
II. Public investment, Percent of GDI			
Baseline	4%	5%	3%
BBR (Default)	4%	6%	3%
SSR (Default)	3%	4%	3%
HR (Default)	6%	11%	3%
BBR (EB)	7%	7%	6%
SSR (EB)	6%	6%	7%
BBR-HR (EB)	11%	15%	8%
SSR-HR (EB)	8%	8%	8%
III. Private investment, Percent of GDI			
Baseline	20%	20%	20%
BBR (Default)	20%	20%	20%
SSR (Default)	20%	20%	20%
HR (Default)	20%	20%	20%
BBR (EB)	20%	20%	20%
SSR (EB)	20%	21%	20%
BBR-HR (EB)	20%	20%	20%
SSR-HR (EB)	20%	21%	20%

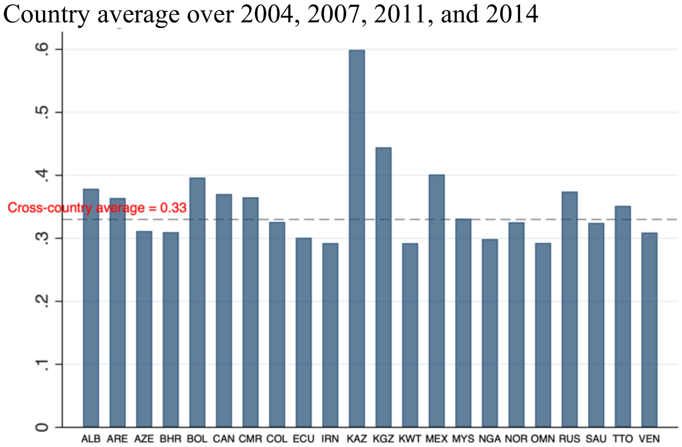
Appendix Table 9: LTGM-NR simulation for Angola: Effects of high discoveries of oil on GDP

	Average growth rate			Real 2010 US$		
	2026-2050	2026-2035	2036-2050	2020	2035	2050
I. GDP per capita						
Baseline	1.1	1.0	1.3	2,890	3,417	4,076
Incremental growth (p.p.):						
BBR (Default)	0.1	0.5	-0.3	//	3,587	4,132
SSR (Default)	0.0	0.5	-0.3	//	3,564	4,106
HR (Default)	0.2	0.9	-0.3	//	3,692	4,238
BBR (EB)	0.1	0.6	-0.2	//	3,618	4,188
SSR (EB)	0.1	0.6	-0.2	//	3,617	4,193
BBR-HR (EB)	0.2	0.9	-0.3	//	3,713	4,270
SSR-HR (EB)	0.2	0.7	-0.1	//	3,634	4,257
II. Non-oil GDP per capita						
Baseline	3.2	3.4	3.3	1,421	2,246	3,558
Incremental growth (p.p.):						
BBR (Default)	0.0	0.0	0.0	//	2,248	3,562
SSR (Default)	0.0	-0.1	0.0	//	2,235	3,530
HR (Default)	0.2	0.3	0.1	//	2,311	3,699
BBR (EB)	0.1	0.1	0.1	//	2,267	3,625
SSR (EB)	0.1	0.1	0.1	//	2,266	3,630
BBR-HR (EB)	0.2	0.4	0.1	//	2,323	3,737
SSR-HR (EB)	0.2	0.1	0.2	//	2,276	3,698
III. Oil GDP per capita						
Baseline	-4.2	-2.4	-5.7	1,469	1,171	518
Incremental growth (p.p.):						
BBR (Default)	0.4	1.5	-0.3	//	1,339	571
SSR (Default)	0.4	1.4	-0.1	//	1,329	576
HR (Default)	0.2	1.8	-0.8	//	1,382	539
BBR (EB)	0.3	1.6	-0.4	//	1,351	562
SSR (EB)	0.3	1.5	-0.4	//	1,351	563
BBR-HR (EB)	0.1	1.9	-1.0	//	1,390	533
SSR-HR (EB)	0.3	1.6	-0.5	//	1,358	558

Appendix Table 10: LTGM-NR simulation for Angola: Effects of high discoveries of oil on investment

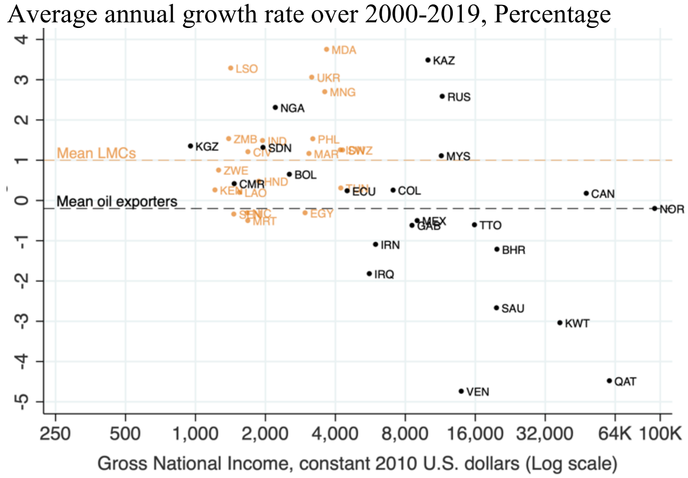
	Percent of GDI		
	ALL 2026-2050	Oil cycle 2026-2035	Post-cycle 2036-2050
I. Investment, Percent of GDI			
Baseline	24%	25%	23%
BBR (Default)	24%	25%	23%
SSR (Default)	23%	24%	23%
HR (Default)	26%	27%	25%
BBR (EB)	25%	25%	24%
SSR (EB)	25%	25%	24%
BBR-HR (EB)	26%	27%	25%
SSR-HR (EB)	25%	26%	25%
II. Public investment, Percent of GDI			
Baseline	3.6%	4.6%	2.9%
BBR (Default)	3.9%	4.9%	3.2%
SSR (Default)	3.5%	4.5%	2.9%
HR (Default)	5.6%	7.1%	4.5%
BBR (EB)	4.5%	5.4%	3.8%
SSR (EB)	4.3%	5.1%	3.8%
BBR-HR (EB)	5.9%	7.4%	4.9%
SSR-HR (EB)	4.9%	5.4%	4.6%
III. Private investment, Percent of GDI			
Baseline	20.0%	20.0%	20.0%
BBR (Default)	20.0%	20.0%	20.0%
SSR (Default)	20.0%	20.0%	20.0%
HR (Default)	20.0%	20.0%	20.0%
BBR (EB)	20.1%	20.1%	20.1%
SSR (EB)	20.3%	20.4%	20.2%
BBR-HR (EB)	20.1%	20.1%	20.1%
SSR-HR (EB)	20.3%	20.4%	20.2%

Appendix Figure 1 Oil rents in large oil exporters,
Country average over 2004, 2007, 2011, and 2014



Source: Authors calculation based on GTAP data

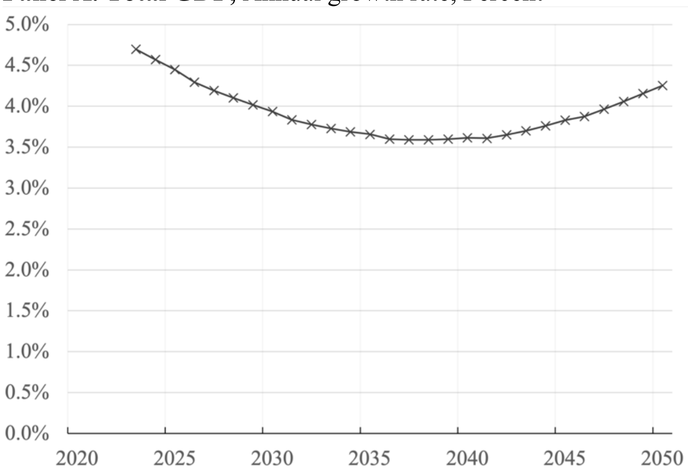
Appendix Figure 2 Total factor productivity,
Average annual growth rate over 2000-2019, Percentage



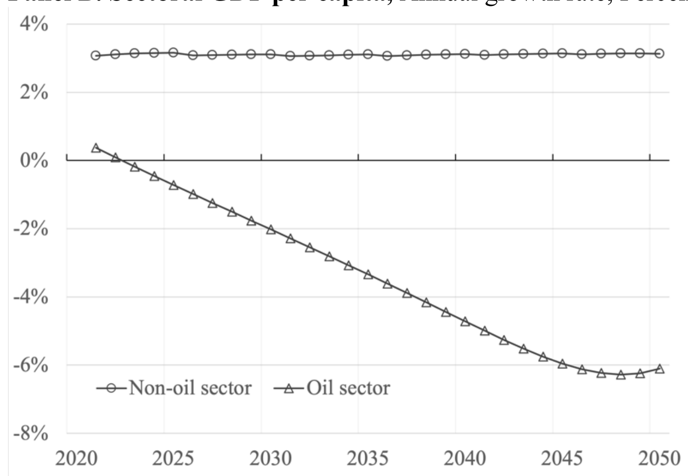
Source: Authors calculation based on PWT10 data

Appendix Figure 3 LTGM-NR baseline simulation for Angola

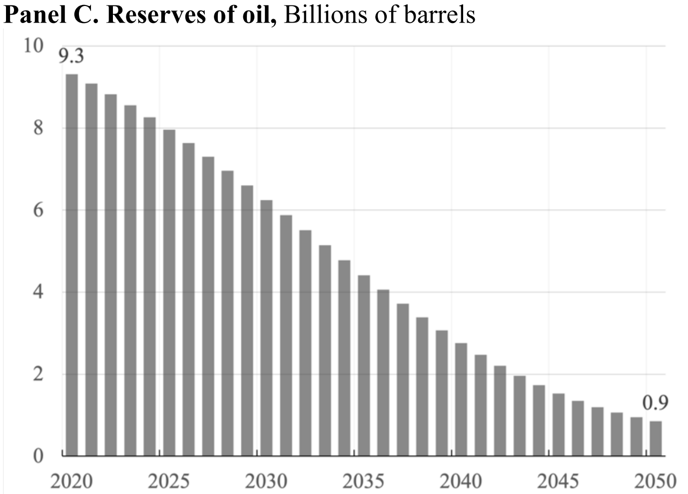
Panel A. Total GDP, Annual growth rate, Percent



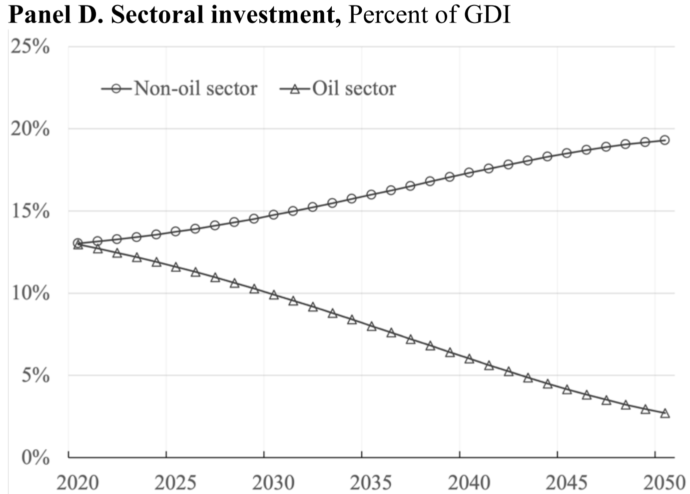
Panel B. Sectoral GDP per capita, Annual growth rate, Percent



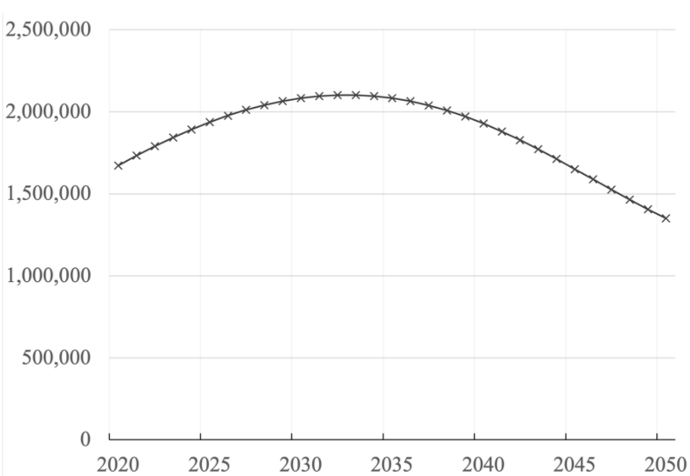
Panel C. Reserves of oil, Billions of barrels



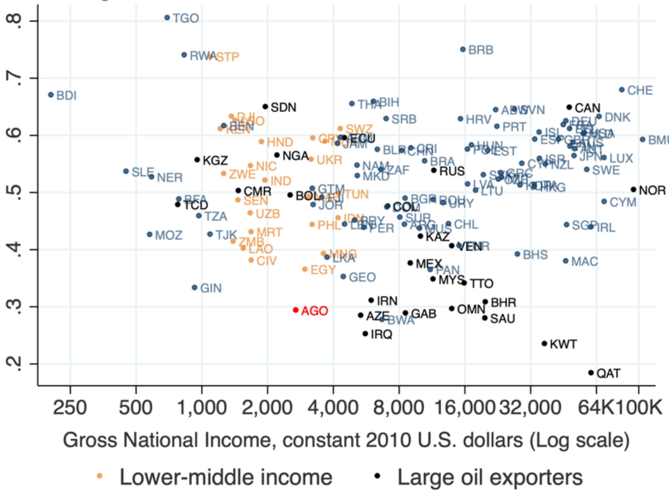
Panel D. Sectoral investment, Percent of GDI



Panel E. Production of oil, Barrels/day

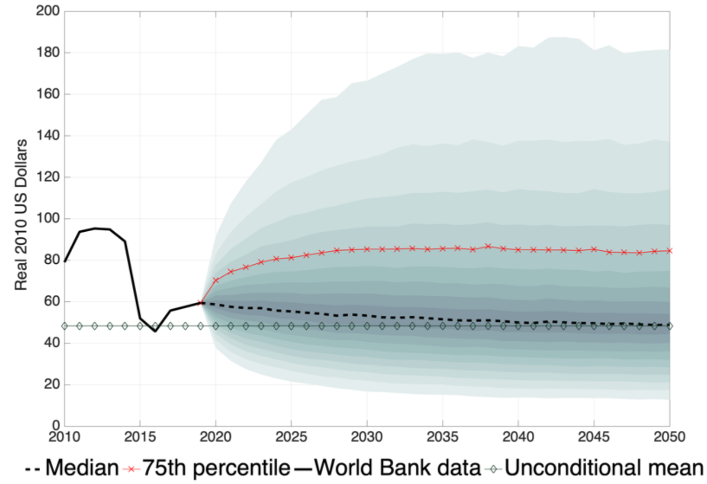


Appendix Figure 4: Labor share,
Average labor compensation share in income over 2000-2019



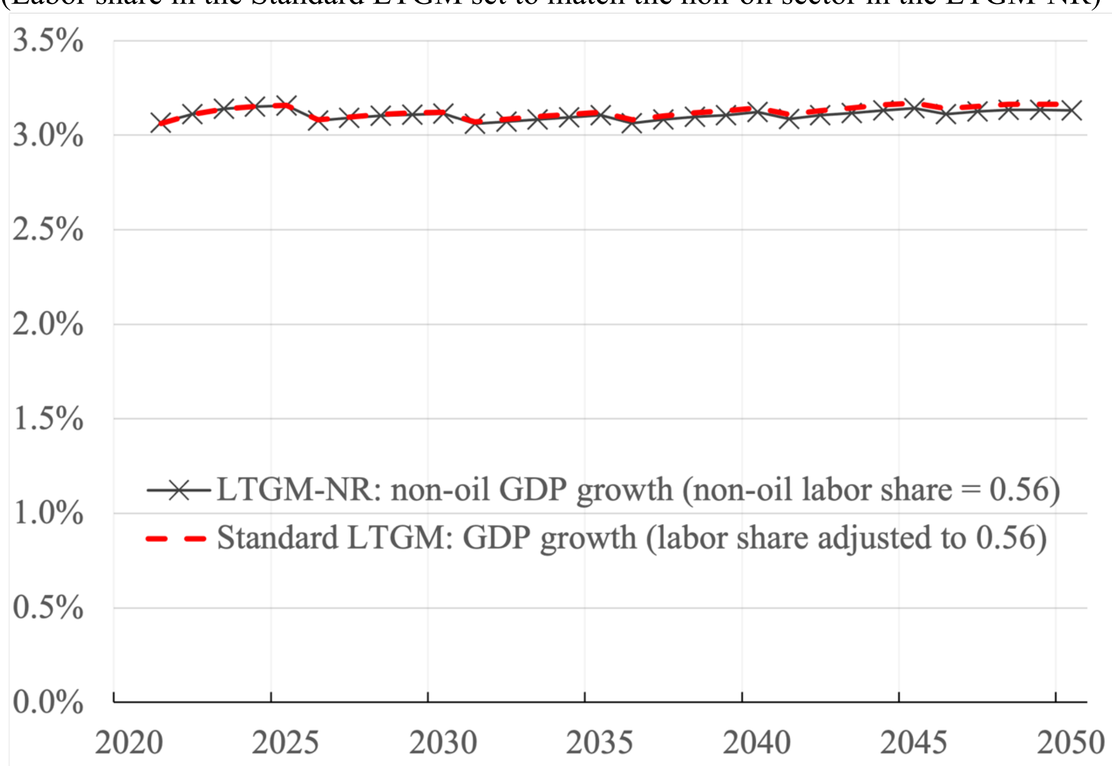
Source: Authors' calculations based on PWT10 data. Outlier Central African Republic was dropped.

Appendix Figure 5: Forecast distribution of oil prices,
Real 2010 U.S. Dollars



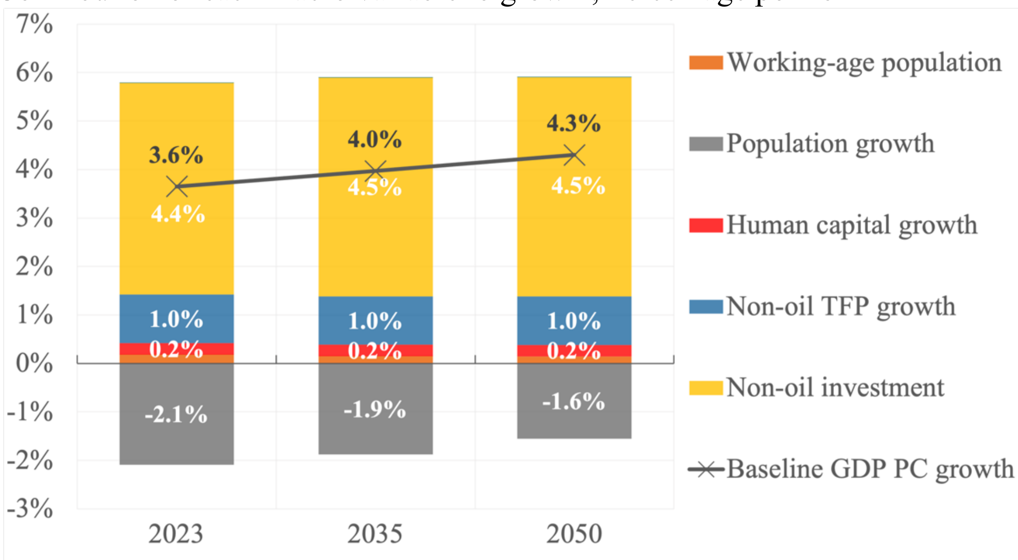
Notes: The model $P_t = \bar{P}^{1-\rho} P_{t-1}^{\rho} e^{u_t}$ is estimated in logs by OLS. The unconditional mean is $\bar{P} e^{\sigma^2/2}$. The forecast distribution is based on 10,000 draws from the estimated OLS process.
Data source: Real Annual Prices from the World Bank Commodity Price Data, The Pink Sheet. We applied the HP-filter on the 75th percentile.

Appendix Figure 6 Simulations for Angola: GDP growth in the LTGM versus non-oil GDP growth in the LTGM-NR
Annual growth rate, Percent
(Labor share in the Standard LTGM set to match the non-oil sector in the LTGM-NR)



Appendix Figure 7 Year-by-year decomposition of baseline GDP per capita growth under the Standard LTGM,

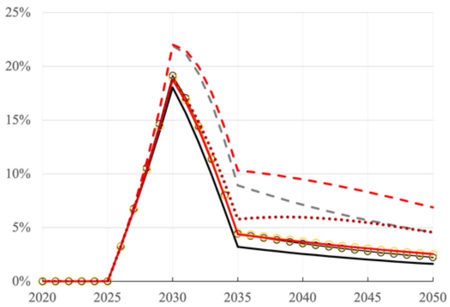
Contribution of each macro variable to growth, Percentage points



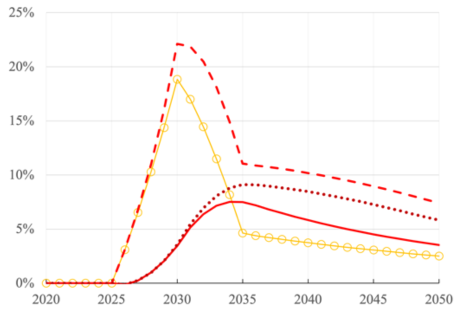
Appendix Figure 8: The effects of oil price shocks in Angola.

Public sector and the external balance

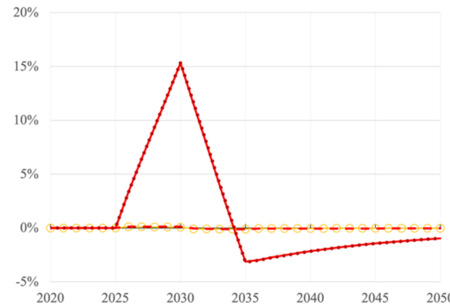
A. Incremental resource revenue,
Percent of baseline GDI



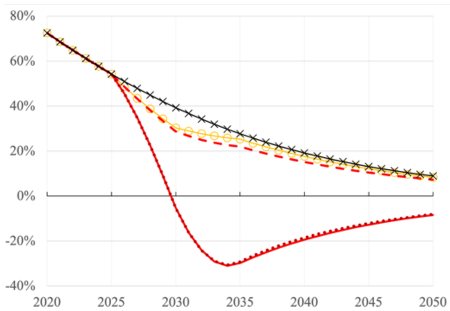
B. Incremental expenditure,
Percent of baseline GDI



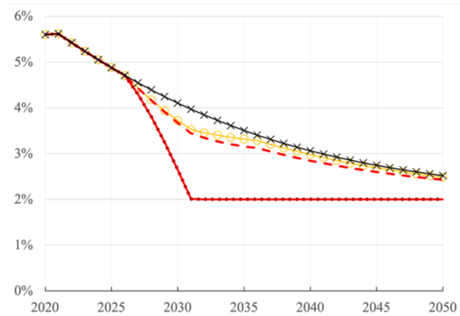
C. Incremental primary balance,
Percent of baseline GDI



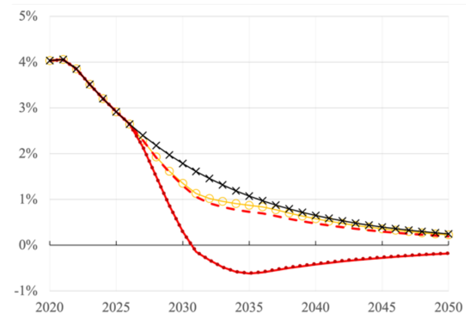
D. External public debt,
Percent of GDI



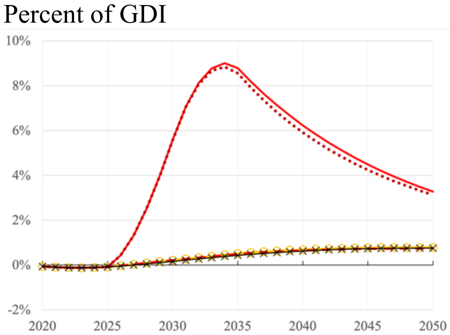
E. Country interest rate,
Percent



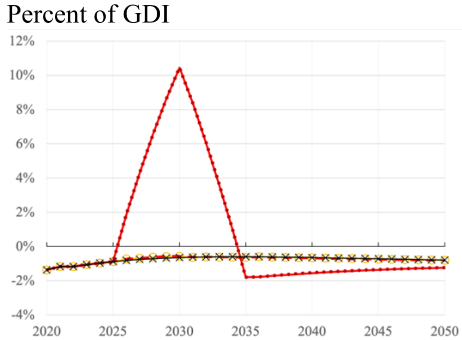
F. Interest expense,
Percent of GDI



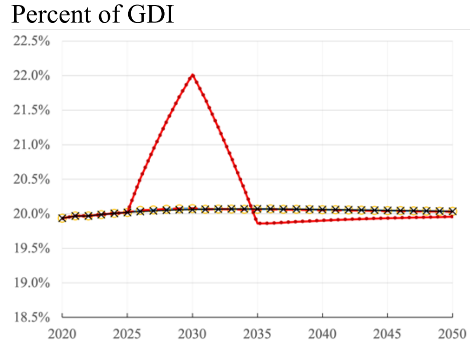
G. External private debt,
Percent of GDI



H. Current account balance,
Percent of GDI



I. Private investment,
Percent of GDI



Appendix Figure 9: The effects of discoveries of oil in Angola
Public sector and the external balance